Mathematics Specialised

LEVEL 4	15 TCE CREDIT POINTS
COURSE CODE	MTS415118
COURSE SPAN	2018 — 2025
READING AND WRITING STANDARD	NO
MATHEMATICS STANDARD	YES
COMPUTERS AND INTERNET STANDARD	NO

This course was delivered in 2022. Use A-Z Courses to find the current version (if available).

Mathematics Specialised is designed for learners with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics, or engineering at university

This course provides opportunities, beyond those presented in Mathematics Methods Level 4, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively.

Course Description

Mathematics Specialised extends the study of functions and calculus, introduces studies in sequences and series, complex numbers, matrices and linear algebra, and deepens the ideas presented in Mathematics Methods, demonstrating their applications in a variety of theoretical and practical contexts.

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Mathematics is also concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Mathematics provides a framework for thinking and a means of communication that is powerful, logical, concise and precise. It impacts upon the daily life of people everywhere and helps them to understand the world in which they live and work.

Mathematics Specialised provides opportunities, beyond those presented in Mathematics Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Learners will be able to appreciate the true nature of mathematics, its beauty and its functionality.

This course contains topics in functions, sequences and series, calculus, matrices and complex numbers that build on and deepen the ideas presented in Mathematics Methods and demonstrate their application in many areas. Complex numbers, mathematics in three dimensions and matrices are introduced.

Mathematics Specialised is designed for learners with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at the tertiary level.

Aims

Mathematics Specialised aims to develop each learner's:

- understanding of concepts and techniques and problem solving ability drawn from algebraic processes, functions and equation study, complex numbers, matrices and linear algebra, and calculus
- reasoning skills in mathematical contexts and in interpreting mathematical information
- capacity to communicate in a concise and systematic manner using mathematical language.

Expectations Of Learners

It is recommended that learners attempting this course will have successfully completed the Mathematics Methods Level 4 course. Assumed knowledge and skills are contained in that course and will be drawn upon in the development of key knowledge and skills in Mathematics Specialised. However, studies in both of these mathematics subjects may be undertaken conjointly with thoughtful sequencing of the areas of study.

Learning Outcomes

On successful completion of this course, learners will be able to:

1. utilise concepts and techniques drawn from sequences and series, complex numbers, matrices and linear algebra, function and equation study, and calculus
2. solve problems using concepts and techniques drawn from algebraic processes, sequences and series, complex numbers, matrices and linear algebra, function and equation study, and calculus
3. apply reasoning skills in the contexts of algebraic processes, sequences and series, complex numbers, matrices and linear algebra, function and equation study, and calculus
4. interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
5. communicate their arguments and strategies when solving problems
6. plan activities and monitor and evaluate their progress
7. use strategies to organise and complete activities, and meet deadlines in the context of mathematics
8. select and use appropriate tools, including computer technology, when solving mathematical problems
9. Additionally, learners will be given opportunities to demonstrate the following, in line with the Australian Curriculum General Capabilities: literacy skills; numeracy skills; information and communication technology skills; critical and creative thinking skills; ethical and intercultural understanding.

Pathways

Mathematics Methods Level 4 provides a pathway into this course. Mathematics Specialised is designed for learners whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level, including science, technology fields, engineering and mathematics (STEM), commerce and economics, and health and social sciences.

Resource Requirements

Learners must have access to calculator algebraic system (CAS) calculators and become proficient in their use. These calculators can be used in all aspects of this course in the development of concepts and as a tool for solving problems. Refer to *'What can I take to my exam?'* and *'Use of calculator policy guidelines (PDF)'* for the current TASC Calculator Policy that applies to Level 3 and 4 courses.

The use of computer software is also recommended as an aid to learning and mathematical development. A range of packages such as, but not limited to: Wolfram *Mathematica*, Microsoft *Excel*, *Autograph*, *Efofex Stat*, and *Graph and Draw* are appropriate for this purpose.

Course Size And Complexity

This course has complexity level of 4.

In general, courses at this level provide theoretical and practical knowledge and skills for specialised and/or skilled work and/or further learning, requiring:

- broad factual, technical and some theoretical knowledge of a specific area or a broad field of work and learning
- a broad range of cognitive, technical and communication skills to select and apply a range of methods, tools, materials and information to:
  - complete routine and non-routine activities
  - provide and transmit solutions to a variety of predictable and sometimes unpredictable problems
- application of knowledge and skills to demonstrate autonomy, judgement and limited responsibility in known or changing contexts and within established parameters.

This Level 4 course has a size value of 15.

Course Requirements

This course is made up of four (4) compulsory areas of study:

- Sequences and series
- Complex numbers
- Matrices and linear algebra
- Calculus.

Each area of study relates to a specific Assessment Criteria (4–8). Assessment Criteria 1–3 apply to all four areas of study.

It is recommended that each area is addressed in the order presented in this document.

It is a requirement of this course that learners study and analyse real-world applications involving the concepts studied in the four major areas of study. This provides learners with mathematical experiences that are much richer than a collection of skills. Learners thereby have the opportunity to observe and make connections between related aspects of the course and the real world and to develop further some important abstract ideas.

Course Content

SEQUENCES AND SERIES (Criterion 4)
Learners will study a range of sequences and series by learning about their properties, meet some of their important uses and begin to understand the ideas of convergence and divergence and develop some methods of proof.

This area of study will include:

- informal applications of absolute value and solution(s) of inequalities
- arithmetic and geometric sequences and series, including the development of formulae for the n^{th} term and the sum to n terms
- the *sum to infinity* of geometric series, and the conditions under which it exists
- the definition of a sequence as a function defined by a subset of natural numbers
- the formal definition of a convergent sequence, and the broad definition of a divergent sequence as one which does not converge
- the formal definition of a sequence which diverges to either positive infinity or negative infinity, and an informal consideration of sequences which oscillate finitely or infinitely
- simple applications of the formal definitions to establish the convergence or divergence of given sequences
- consideration of the special sequences $\left\{x^n\right\}$ and $\left\{\left(1+\frac{1}{n}\right)^n\right\}$
- recursive definitions and sigma notation
- the nature of proof
- proof by mathematical induction applied to series (where the n^{th} term is given)
- the results $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$, $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$ established by a *method of differences* or by mathematical induction, and the sums of series utilising these results
- the *method of differences*, confined to series such as $\sum_{r=1}^{n} U_r$ and in simple cases $\sum_{r=1}^{\infty} U_r$, where it is explicitly clear that U_r can be expressed as a difference of two terms of a sequence V_r
- MacLaurin series for simple functions such as $(1+x)^n$, $\sin ax$, $\cos ax$, e^{ax} and $\ln(x+1)$ with an informal consideration of interval of convergence.

<u>Area Work Requirements</u>
The minimum work requirements for this area of study, assessing Criterion 4, will include learners completing one (1) major and two (2) minor assessments where they will find solutions to mathematical questions/problems that may:

- be routine, analytical, and/or interpretive
- be posed in familiar and unfamiliar contexts
- require a discerning use of electronic technology.

Also see Work Requirements – Applications below.

COMPLEX NUMBERS (Criterion 8)
Learners will be introduced to a different class of numbers and understand that such numbers can be represented in several ways and that their use allows factorisation to be carried out more fully than was previously possible.

This area of study will include:

- the Cartesian form of a complex number $a + ib$, its real and imaginary parts, and fundamental operations involving complex numbers in this form
- representation of a complex number on the Argand plane as a point or vector
- the polar form of a complex number $r(\cos\theta + i\sin\theta)$ or $r(cis\theta)$, including conversion between Cartesian form and polar form
- using Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$, to link polar and exponential form (using MacLaurin series or differential equations)
- the modulus $|z|$, argument $arg(z)$, and principal argument $Arg(z)$ of a complex number
- multiplication and division of complex numbers in polar form, including the use of De Moivre's theorem
- the conjugate $\bar{z}$ of a complex number, and the result that $z.\bar{z} = |z|^2$
- De Moivre's theorem and its proof for rational exponents
- application of De Moivre's theorem to solving equations of the form $z^n = p$, where n is a positive integer, including sketching the solution sets on the Argand plane
- expression of a polynomial as a sum of geometric series to assist in the factorisation of; the polynomial (e.g. solve $z^6 - z^4 + z^2 - 1 = 0$)

- applications of De Moivre's theorem and the conjugate root theorem to factorise a polynomial into linear and real (quadratic) factors, and to simplifying expressions such as $(1 + i)^5\left(\sqrt{3} - i\right)^4$
- informal treatment of the *Fundamental Theorem of Algebra*
- regions of the Argand plane satisfying simple functions – straight lines, polynomials, hyperbola, truncus and square roots as well as circles and ellipses and combinations of these examples:
  $|z| \geq 4,\ \frac{\pi}{6} < Arg(z) \leq \frac{2\pi}{3},\ [lm(z - i)]^2 + [Re(z + i)]^2 = 1$

<u>Area Work Requirements</u>
The minimum work requirements for this area of study, assessing Criterion 8, will include learners completing one (1) major and two (2) minor assessments where they will find solutions to mathematical questions/problems that may:

- be routine, analytical, and/or interpretive
- be posed in familiar and unfamiliar contexts
- require a discerning use of electronic technology.

Also see Work Requirements – Applications below.

MATRICES AND LINEAR ALGEBRA (Criterion 5)
Learners will be introduced to new mathematical structures and appreciate some of the ways in which these structures can be put to use. Trigonometric identities will be used to develop ideas in matrices and linear transformations.

This area of study will include:

- addition and multiplication of matrices, including the concepts of identities, inverses, associativity and commutativity
- determinant of a 2 x 2 matrix, and the idea of singularity or non-singularity
- solutions of two equations in two unknowns or three equations in three unknowns, including the process of Gauss-Jordan reduction, and the use of technology to solve larger systems (applications should be included)
- interpret solutions, where applicable, to Gauss-Jordan systems as points, lines (in parametric or symmetric form) or planes in 3 dimensional space including:
  - planes and their equations in 3 dimensions; sketching planes in 3 dimensions using intercepts with x, y and z axes
  - demonstrating whether a line or point is embedded in a given plane
  - recognising when two planes are parallel
- the definition of a linear transformation (with an emphasis on non-singular transformations), and its representation by a matrix
- the image of a point, the *unit square*, a straight line, a circle or a curve under a non-singular linear transformation
- a study of dilation, shear, rotation (about the origin) and reflection (in the line $y = (\tan\alpha).\ x$); transformations and composites of these
- the relationship between the determinant of a matrix and the area of an image
- composition of transformations used to develop the addition theorems for $\cos(A \pm B)$, $\sin(A \pm B)$ and $\tan(A \pm B)$
- the *double angle* formulae for $\cos 2A$, $\sin 2A$ and $\tan 2A$

<u>Area Work Requirements</u>
The minimum work requirements for this area of study, assessing Criterion 5, will include learners completing one (1) major and two (2) minor assessments where they will find solutions to mathematical questions/problems that may:

- be routine, analytical, and/or interpretive
- be posed in familiar and unfamiliar contexts
- require a discerning use of electronic technology.

Also see Work Requirements – Applications below.

CALCULUS (Criteria 6 and 7)
Learners will extend their existing knowledge and understanding of this vital branch of mathematics by developing a greater capacity for integrating functions and by introducing simple differential equations and their uses.

This section of the course develops and extends the ideas introduced in the Mathematics Methods course.

This area of study will include:

PART 1: (Criterion 6)

- implicit differentiation and its use in finding tangents and normals to curves
- a review of rules for differentiating functions as described in Mathematics Methods, derivatives of inverse trigonometric functions, a^x and $\log_a x$, and compositions of these functions
- application of first and second derivatives to curve sketching, including stationary points, points of inflection and changes in concavity
- review of the *Fundamental Theorem of Calculus*
- properties of definite integrals:
  - $\int_a^a f(x)dx = 0$
  - $\int_a^b f(x)dx = -\int_b^a f(x)dx$
  - $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$
  - $\int_a^b k.f(x)dx = k\int_a^b f(x)dx$, for k constant
  - $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
- applications of definite integrals to finding areas under or between curves and to finding volumes of solids of revolution about either the x-axis or the y-axis

PART 2: (Criterion 7)

- techniques of integration:
  - integrals of functions with linear arguments
  - trigonometric identities (*double angle* formulae and *products to sums*)
  - inverse trigonometric identities, e.g. $\int \frac{9}{1 + 4x^2} dx$
  - partial fractions
  - substitution, $u = g(x)$ to integrate expressions of the form $f(g(x))g'(x)$
  - linear substitution, e.g. $\int x(3x - 1)^6 dx$
  - integration by parts
  - differentiating and then integrating, e.g. find $\frac{d}{dx}(x\ln x)$ and hence find $\int \ln x dx$
- first order linear differential equations of the type:
  - $\frac{dy}{dx} = f(x)$ $\frac{dy}{dx} = f(y)$
  - $f(x) + g(y)\frac{dy}{dx} = 0$ $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$
- applications of differential equations, but not including input/output problems.

Area Work Requirements

The minimum work requirements for these areas of study, assessing Criterion 6 and Criterion 7, will include learners completing two (2) major and four (4) minor assessments where they will find solutions to mathematical questions/problems that may:

- be routine, analytical, and/or interpretive
- be posed in familiar and unfamiliar contexts
- require a discerning use of electronic technology.

Also see Work Requirements – Applications below.

Work Requirements

APPLICATIONS

In their study of this course, learners will investigate a minimum of two applications. It is recommended that the selected applications relate to different course areas. Such extended problems, investigations and applications of technology provide opportunities to reinforce many of the skills and concepts studied in this course.

Examples include the following:

- Koch snowflake
- calculating the expected value and variance of various shaped discrete distributions, e.g. the 'tent' function/map
- graphs of functions in polar form
- graphs of functions in parametric form
- coding with matrices
- matrices in the real world, such as Markov chains or Leslie Matrices
- affine transformations in two or three dimensions
- vectors in the two dimensional plane, these could include magnitude and components of vectors, dot product and the resolution of a vector into two components
- length of a curve
- the trapezoidal rule and its use to approximate the area under a curve of a non-integral function
- mathematical induction involving inequalities or recursively defined sequences and series
- monotonicity and boundedness of sequences
- Mandelbrot set
- simple harmonic motion
- Coriolis component of acceleration (as affects our weather)
- spring–mass systems
- balancing of a 6-cylinder petrol engine
- centripetal acceleration (derivative of equation)
- kinematics
- volumes of common three dimensional objects, e.g. bottles, doughnuts
- examine slope (direction or gradient) fields of first order differential equations
- formulate differential equations including the logistic equation that will arise in, for example, chemistry, biology and economics, is situations where rates are involved.

SUMMARY

In total, the minimum work requirements for this course are:

- at least five (5) major assessments and ten (10) minor assessments covering each of the content Criteria 4, 5, 6 ,7 and 8
- two (2) (in total) investigations into some applications of content areas of this course. Each is to be assessed against Criteria 1, 2 and 3 and against one of the content Criteria 4, 5, 6, 7 or 8.

Assessment

Criterion-based assessment is a form of outcomes assessment that identifies the extent of learner achievement at an appropriate end-point of study. Although assessment – as part of the learning program – is continuous, much of it is formative, and is done to help learners identify what they need to do to attain the maximum benefit from their study of the course. Therefore, assessment for summative reporting to TASC will focus on what both teacher and learner understand to reflect end-point achievement.

The standard of achievement each learner attains on each criterion is recorded as a rating 'A', 'B', or 'C', according to the outcomes specified in the standards section of the course.

A 't' notation must be used where a learner demonstrates any achievement against a criterion less than the standard specified for the 'C' rating.

A 'z' notation is to be used where a learner provides no evidence of achievement at all.

Providers offering this course must participate in quality assurance processes specified by TASC to ensure provider validity and comparability of standards across all awards. For further information, see quality assurance and assessment processes.

Internal assessment of all criteria will be made by the provider. Providers will report the learner's rating for each criterion to TASC.

TASC will supervise the external assessment of designated criteria which will be indicated by an asterisk (*). The ratings obtained from the external assessments will be used in addition to internal ratings from the provider to determine the final award.

Quality Assurance Process

The following processes will be facilitated by TASC to ensure there is:

- a match between the standards of achievement specified in the course and the skills and knowledge demonstrated by learners
- community confidence in the integrity and meaning of the qualification.

Process – TASC gives course providers feedback about any systematic differences in the relationship of their internal and external assessments and, where appropriate, seeks further evidence through audit and requires corrective action in the future.

External Assessment Requirements

The external assessment for this course will comprise:

- a three hour written examination assessing Criteria 4, 5, 6, 7 and 8.

For further information, see Exams and Assessment for the current external assessment specifications and guidelines for this course.

Criteria

The assessment for Mathematics Specialised Level 4 will be based on the degree to which the learner can:

1. communicate mathematical ideas and information
2. apply mathematical reasoning and strategies in problem solving situations
3. use resources and organisational strategies
4. solve problems and use techniques involving finite and infinite sequences and series*
5. solve problems and use techniques involving matrices and linear algebra*
6. use differential calculus and apply integral calculus to areas and volumes*
7. use techniques of integration and solve differential equations*
8. solve problems and use techniques involving complex numbers*

Note: * denotes criteria that are both internally and externally assessed

Standards

Criterion 1: communicate mathematical ideas and information

The learner:

Rating A	Rating B	Rating C
presents work that conveys a logical line of reasoning that has been followed between question and answer	presents work that conveys a line of reasoning that has been followed between question and answer	presents work that shows some of the mathematical processes that have been followed between question and answer
consistently uses mathematical conventions and symbols correctly	generally uses mathematical conventions and symbols correctly	uses mathematical conventions and symbols (There may be some errors or omissions in doing so.)
presents work with the final answer clearly identified, and articulated in terms of the question as required	presents work with the final answer clearly identified	presents work with the final answer apparent
uses correct units and includes them in an answer for routine and non-routine problems	uses correct units and includes them in an answer for routine problems	uses correct units and includes them in an answer for routine problems
presents detailed tables, graphs and diagrams that convey accurate meaning and precise information	presents detailed tables, graphs and diagrams that convey clear meaning	presents tables, graphs and diagrams as directed
ensures a degree of accuracy appropriate to task is maintained – including the use of exact values – communicated throughout a problem	determines and works to a degree of accuracy appropriate to task, including the use of exact values where required	works to a degree of accuracy appropriate to tasks, as directed

Criterion 2: apply mathematical reasoning and strategies in problem solving situations

The learner:

Rating A	Rating B	Rating C
selects and applies an appropriate strategy (where several may exist) to solve routine and non-routine problems in a variety of contexts	selects and applies an appropriate strategy to solve routine and simple non-routine problems	identifies an appropriate strategy to solve routine problems
interprets solutions to routine and non-routine problems	interprets solutions to routine and simple non-routine problems	describes solutions to routine problems
explains the reasonableness of results and solutions to routine and non-routine problems	describes the reasonableness of results and solutions to routine problems	describes the appropriateness of the results of calculations
identifies and describes limitations of presented models, and as applicable, explores the viability of possible alternative models	identifies and describes limitations of presented models	identifies limitations of simple models
uses available technological aids in familiar and unfamiliar contexts	chooses to use available technological aids when appropriate to solve routine problems	uses available technological aids to solve routine problems
explores the use of technology in familiar and unfamiliar contexts	explores the use of technology in familiar contexts	
constructs and solves problems derived from routine and	constructs and solves problems derived	

non-routine scenarios	from routine scenarios	

Criterion 3: use resources and organisational strategies

The learner:

Rating A	Rating B	Rating C
uses planning tools and strategies to achieve and manage activities within proposed times	uses planning tools to achieve objectives within proposed time	uses planning tools, with prompting, to achieve objectives within proposed times
divides tasks into appropriate sub-tasks in multiple step operations	divides a task into appropriate sub-tasks	divides a task into sub-tasks
selects strategies and formulae to successfully complete routine and non-routine problems	selects from a range of strategies and formulae to successfully complete routine and non-routine problems	selects from a range of strategies and formulae to complete routine problems
plans timelines and monitors and analyses progress towards meeting goals, making adjustments as required	plans timelines and monitors progress towards meeting goals	monitors progress towards meeting goals
addresses all of the required elements of a task with a high degree of accuracy	addresses the elements of required tasks	addresses most elements of required tasks
plans future actions, effectively adjusting goals and plans where necessary	plans future actions, adjusting goals and plans where necessary	uses prescribed strategies to adjust goals and plans where necessary

Criterion 4: solve problems and use techniques involving finite and infinite sequences and series

This criterion is both internally and externally assessed.

Rating 'A'
In addition to the standards for a C and a B rating, the learner:

Rating 'B'
In addition to the standards for a C rating, the learner:

Rating 'C'
The learner:

Rating A	Rating B	Rating C
determines whether a given number is a term of a given sequence	determines the n^{th} term of a sequence	determines the next few terms of a sequence, e.g. $\left\{u_n\right\} = \frac{1}{2 \times 3}, -\frac{5}{3 \times 4}, \frac{9}{4 \times 5}, \ldots$
determines the sum to n terms of arithmetic and geometric progressions in questions of a greater complexity	determines the sum to n terms of recursively defined arithmetic and geometric progressions, e.g. find the sum to n terms of $u_1 = xy^2, u_{n+1} = \frac{x}{y}u_n$	determines the sum to n terms of arithmetic and geometric progressions, e.g. find the sum to n terms of $\left\{u_n\right\} = 3k, 7k, 11k, \ldots$
interprets the conditions required for the infinite sum of geometric series to exist, for series of a greater complexity	interprets the conditions required for the infinite sum of a geometric series to exist	determines the infinite sum of simple geometric series and states the condition required for it to exist

		states the formal definitions of a sequence and requirements for a sequence to converge and diverge to $\pm\infty$
states and prove convergence or divergence of a sequence where a variety of algebraic techniques are required	states and proves convergence or divergence of a sequence and interprets results given K or ε	states and proves convergence or divergence of a sequence in simple cases
determines sums of series to a given number or an infinite number of terms using standard results in cases where there may be a number of sub-series	determines sums of series to n terms using standard results in cases with a greater complexity	determines sums of series to n terms using standard results in simple cases
uses mathematical induction to show that a series has a given sum to n terms, where the n^{th} term is given, and interpret the results	uses mathematical induction to show that a series has a given sum to n terms, where the n^{th} term is given	
uses a *method of differences* to sum routine and non-routine series	uses a *method of differences* to sum routine series	
determines MacLaurin series for functions of the prescribed type, and integrates, differentiates or substitutes to find other series or approximations	determines MacLaurin series for functions of the prescribed type	

Criterion 5: solve problems and use techniques involving matrices and linear algebra

This criterion is both internally and externally assessed.

Rating 'A'
In addition to the standards for a C and a B rating, the learner:

Rating 'B'
In addition to the standards for a C rating, the learner:

Rating 'C'
The learner:

Rating A	Rating B	Rating C
use properties of matrices to determine or demonstrate results, e.g. show $(AB)^{-1} = B^{-1}A^{-1}$, where A and B are non-singular matrices	recognises and applies properties of matrices in less routine cases	recognises and applies properties of matrices in routine cases, e.g. find a and b given that $\begin{pmatrix} 1 & b \\ a & 1 \end{pmatrix}\begin{pmatrix} 2a & a \\ a & 1 \end{pmatrix} = \begin{pmatrix} 10 & 5 \\ 10 & 5 \end{pmatrix}$
solves equations involving matrices where the solution is expressed in a general form	solves equations involving matrices that may require the use of an inverse matrix	solves routine equations involving matrices
uses Gauss-Jordan reduction to solve systems of two equations in two unknowns, or two or three equations in three unknowns where one or more element(s) in the matrix is designated symbolically	uses Gauss-Jordan reduction to solve systems of two equations in two unknowns, or two or three equations in three unknowns where no solutions or an infinite number of solutions may exist	uses Gauss-Jordan reduction to solve systems of two equations in two unknowns, or three equations in three unknowns where, if a solution exists, it is unique
determines the equation of a plane passing through three points using Gauss-Jordan reduction	interprets solutions to systems of equations as points or lines (in parametric or symmetric form) in 3D space	interprets equations as lines or planes in 3 dimensional space
solves non-routine problems involving lines	demonstrate whether two or three planes	demonstrate whether a line or a

and planes in 3 dimensional space	are parallel using Gauss-Jordan reduction	point is embedded in a plane
applies techniques in linear transformations to solve routine and non-routine problems in a variety of contexts, e.g. show that the image of a circle when rotated by θ radians is also a circle	uses techniques in composite linear transformations, e.g. find the image of the circle $x^2 + y^2 = 1$ when it is dilated by a factor of 4 parallel to the y-axis and rotated clockwise through $\frac{\pi}{2}$	uses techniques in linear transformations, e.g. find the image of the circle $x^2 + y^2 = 1$ when it is dilated by a factor of 4 parallel to the y-axis
	applies composite transformations to develop addition theorems and associated results	
	uses area-related information to make inferences about determinants or matrix elements in order to solve problems	applies determinants to areas of images, e.g. if the linear transformation $T:(x, y) \rightarrow (x + y, x - y)$ is applied to the unit square, find the resulting area

Criterion 6: use differential calculus and apply integral calculus to areas and volumes

This criterion is both internally and externally assessed.

Rating 'A'
In addition to the standards for a C and a B rating, the learner:

Rating 'B'
In addition to the standards for a C rating, the learner:

Rating 'C'
The learner:

Rating A	Rating B	Rating C
	determines the derivatives of a^x and $\log_a x$, and of inverse trigonometric functions, including products and compositions of these functions	determines the derivatives of a^x and $\log_a x$, and of inverse trigonometric functions
finds tangents and normals of routine and non-routine explicit or implicit functions	finds tangents and normals of routine explicit or implicit composite functions	applies techniques in derivatives of explicit and implicit functions to find tangents and normals
finds and classifies stationary points and non-stationary points of inflection of functions of greater complexity and interprets the answer	finds and classifies stationary points and stationary and non-stationary points of inflection of functions including basic trigonometric functions	finds and classifies stationary points and points of inflection of functions, excluding trigonometric functions
determines areas and volumes in routine and non-routine cases, including compound (or composite) areas and interprets the answer	determines areas and volumes in routine cases including finding the area between a curve and the y-axis	applies properties of definite integrals to calculate areas and volumes
uses the concavity of a function in sketching its curve (where the function has features of a greater complexity)	determines the domain for particular concavity of functions	uses points of inflection to determine the concavity of functions in given domains

Criterion 7: use techniques of integration and solve differential equations

This criterion is both internally and externally assessed.

Rating 'A'
In addition to the standards for a C and a B rating, the learner:

Rating 'B'
In addition to the standards for a C rating, the learner:

Rating 'C'
The learner:

Rating A	Rating B	Rating C
resolves expressions into partial fractions and integrates rational functions with non-linear factors	resolves expressions into partial fractions and integrates rational functions with repeating linear factors	resolves expressions into partial fractions and integrates proper rational functions with non-repeating linear factors
manipulates trigonometric identities to determine integrals in cases of greater complexity, e.g. find $\int \sin^3 3x dx$	uses trigonometric identities to determine integrals in cases of more complexity, e.g. find $\int \frac{x^2}{x^2+1} dx$	uses trigonometric identities to determine integrals, e.g. find $\int \frac{9}{1+4x^2} dx$
determines integrals involving manipulations and substitutions of greater complexity , e.g. find $\int \frac{x^2}{(2x-3)^3} dx$	determines integrals involving substitutions and linear substitutions of more complexity, e.g. find $\int \frac{3x+1}{\sqrt{4x-1}} dx$	determines integrals involving simple substitutions and linear substitutions, e.g. find $\int x(4x-1)^3 dx$
determines integrals involving integration by parts more than once	determines integrals involving integration by parts once	
solves homogeneous equations	solves differential equations involving separable variables	solves differential equations involving a function of x or y
establishes and applies differential equations to solve practical problems	applies given differential equations to solve practical problems	

Criterion 8: solve problems and use techniques involving complex numbers

This criterion is both internally and externally assessed.

Rating 'A'
In addition to the standards for a C and a B rating, the learner:

Rating 'B'
In addition to the standards for a C rating, the learner:

Rating 'C'
The learner:

Rating A	Rating B	Rating C
	infers equations from real and imaginary parts of a complex equation and solves associated problems	uses the notation associated with complex numbers and performs basic operations
converts between rectangular and polar forms when solving problems of a greater complexity, e.g. simplify $\frac{\left(i\sqrt{3}+1\right)^2}{\left(-\sqrt{3}+i\right)^4(-1-i)^4}$	converts between rectangular and polar forms when solving basic problems, e.g. demonstrate that $1-i=\sqrt{2}cis\left(-\frac{\pi}{4}\right)=\sqrt{2}e^{\left(i,-\frac{\pi}{4}\right)}$	converts between rectangular and polar forms
locates points, lines, curves or regions on	locates points, lines, curves or regions	locates points, lines, curves or regions on

the Argand plane defined by one or more conditions including points at intersections of boundaries of regions	on the Argand plane defined by more than one simple condition	the Argand plane defined by one simple condition
manipulates and solves simple polynomial equations with complex roots and factorises associated polynomials	solves simple polynomial equations with complex roots	solves simple linear, quadratic or simultaneous equations with complex roots; finds n^{th} roots of complex numbers, where n is a positive integer
solves polynomial equations, where the terms are in geometric progression, in non-routine situations	solves polynomial equations, where the terms are in geometric progression, in routine situations	
applies conjugate root theorem to solve degree 4 polynomials with real coefficients	applies conjugate root theorem to solve degree 3 polynomials with real coefficients	

Qualifications Available

Mathematics Specialised Level 4 (with the award of):

EXCEPTIONAL ACHIEVEMENT (EA)
HIGH ACHIEVEMENT (HA)
COMMENDABLE ACHIEVEMENT (CA)
SATISFACTORY ACHIEVEMENT (SA)
PRELIMINARY ACHIEVEMENT (PA)

Award Requirements

The final award will be determined by the Office of Tasmanian Assessment, Standards and Certification from 13 ratings (8 from the internal assessment, 5 from external assessment).

The minimum requirements for an award in Mathematics Specialised Level 4 are as follows:

EXCEPTIONAL ACHIEVEMENT (EA)
11 ‘A’ ratings, 2 ‘B’ ratings (4 ‘A’ ratings and 1 ‘B’ rating in the external assessment)

HIGH ACHIEVEMENT (HA)
5 ‘A’ ratings, 5 ‘B’ ratings, 3 ‘C’ ratings (2 ‘A’ ratings, 2 ‘B’ ratings and 1 ‘C’ rating in the external assessment)

COMMENDABLE ACHIEVEMENT (CA)
7 ‘B’ ratings, 5 ‘C’ ratings (2 ‘B’ ratings and 2 ‘C’ ratings in the external assessment)

SATISFACTORY ACHIEVEMENT (SA)
11 ‘C’ ratings (3 ‘C’ ratings in the external assessment)

PRELIMINARY ACHIEVEMENT (PA)
6 ‘C’ ratings

A learner who otherwise achieves the ratings for a CA (Commendable Achievement) or SA (Satisfactory Achievement) award but who fails to show any evidence of achievement in one or more criteria (‘z’ notation) will be issued with a PA (Preliminary Achievement) award.

Course Evaluation

The Department of Education’s Curriculum Services will develop and regularly revise the curriculum. This evaluation will be informed by the experience of the course’s implementation, delivery and assessment.

In addition, stakeholders may request Curriculum Services to review a particular aspect of an accredited course.

Requests for amendments to an accredited course will be forward by Curriculum Services to the Office of TASC for formal consideration.

Such requests for amendment will be considered in terms of the likely improvements to the outcomes for learners, possible consequences for delivery and assessment of the course, and alignment with Australian Curriculum materials.

A course is formally analysed prior to the expiry of its accreditation as part of the process to develop specifications to guide the development of any replacement course.

Course Developer

The Department of Education acknowledges the significant leadership of Andrew Woolley and Lance Coad in the development of this course. The ongoing Mathematics educational legacy and the past leadership of Neville Windsor is also acknowledged.

Expectations Defined By National Standards In Content Statements Developed by ACARA

The statements in this section, taken from documents endorsed by Education Ministers as the agreed and common base for course development, are to be used to define expectations for the meaning (nature, scope and level of demand) of relevant aspects of the sections in this document setting out course requirements, learning outcomes, the course content and standards in the assessment.

For the content areas of Mathematics Methods, the proficiency strands – Understanding; Fluency; Problem Solving; and Reasoning – build on learners' learning in F-10 Australian Curriculum: *Mathematics*. Each of these proficiencies is essential, and all are mutually reinforcing. They are still very much applicable and should be inherent in the five areas of study.

Specialist Mathematics

Unit 2 – Topic 1: Trigonometry

The basic trigonometric functions:

- find all solutions of $f(a(x - b)) = c$ where f is one of $\sin$, $\cos$ or $\tan$ (ACMSM042)
- graph functions with rules of the form $y = f(a(x - b))$ where f is one of $\sin$, $\cos$ or $\tan$ (ACMSM043)

Compound angles:

- prove and apply the angle sum, difference and double angle identities (ACMSM044)

The reciprocal trigonometric functions, secant, cosecant and cotangent:

- define the reciprocal trigonometric functions, sketch their graphs, and graph simple transformations of them (ACMSM045)

Trigonometric identities:

- prove and apply the Pythagorean identities (ACMSM046)
- prove and apply the identities for products of sines and cosines expressed as sums and differences (ACMSM047)
- convert sums $a\cos x + b\sin x$ or $R\cos(x \pm \alpha)$ or $R\sin(x \pm \alpha)$, and apply these to sketch graphs, solve equations of the form $a\cos x + b\sin x = c$, and solve problems (ACMSM048)
- prove and apply other trigonometric identities such as $\cos 3x = 4\cos^3 x - 3\cos x$ (ACMSM049)

Applications of trigonometric functions to model periodic phenomena:

- model periodic motion using sine and cosine functions and understand the relevance of the period and amplitude of these functions in the model (ACMSM050)

Unit 2 – Topic 2: Matrices

Matrix arithmetic:

- understand the matrix definition and notation (ACMSM051)
- define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and inverse (ACMSM052)
- calculate the determinant and inverse of 2×2 matrices and solve matrix equations of the form $AX = B$, where A is a 2×2 matrix, and X and B are column vectors (ACMSM053)

Transformations in the plane:

- translations and their representation as column vectors (ACMSM054)
- define and use basic linear transformations: dilations of the form $(x, y) \to \left(\lambda_1 x, \lambda_2 y\right)$, rotations about the origin and reflection in a line which passes through the origin, and the representations of these transformations by 2×2 matrices (ACMSM055)
- apply these transformations to points in the plane and geometric objects (ACMSM056)
- define and use composition of linear transformations and the corresponding matrix products (ACMSM057)
- define and use inverses of linear transformations and the relationship with the matrix inverse (ACMSM058)
- examine the relationship between the determinant and the effect of a linear transformation on area (ACMSM059)
- establish geometric results by matrix multiplications, e.g. show that the combined effect of two reflections in lines through the origin is a rotation (ACMSM060)

Unit 2 – Topic 3: Real and Complex Numbers

Rational and irrational numbers:

- express rational numbers as terminating or eventually recurring decimals and vice versa (ACMSM062)

An introduction to proof by mathematical induction:

- understand the nature of inductive proof including the 'initial statement' and inductive step (ACMSM064)
- prove results for sums, such as $1 + 4 + 9 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer n (ACMSM065)

Complex numbers:

- define the imaginary number i as a root of the equation $x^2 = -1$ (ACMSM067)
- use complex numbers in the form $a + bi$ where a and b are the real and imaginary parts (ACMSM068)
- determine and use complex conjugates (ACMSM069)
- perform complex-number arithmetic: addition, subtraction, multiplication and division (ACMSM070)

The complex plane:

- consider complex numbers as points in a plane with real and imaginary parts as Cartesian coordinates (ACMSM071)
- understand and use location of complex conjugates in the complex plane (ACMSM073)

Roots of equations:

- use the general solution of real quadratic equations (ACMSM074)
- determine complex conjugate solutions of real quadratic equations (ACMSM075)
- determine linear factors of real quadratic polynomials (ACMSM076)

Unit 3 – Topic 1: Complex Numbers

Cartesian forms:

- review real and imaginary parts $Re(z)$ and $Im(z)$ of a complex number z (ACMSM077)
- review Cartesian form (ACMSM078)
- review complex arithmetic using Cartesian forms (ACMSM079)

Complex arithmetic using polar form:

- use the modulus $|z|$ of a complex number z and the argument $Arg(z)$ of a non-zero complex number z, and prove basic identities involving modulus and argument (ACMSM080)
- convert between Cartesian and polar form (ACMSM081)
- define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these (ACMSM082)
- prove and use De Moivre's theorem for integral powers (ACMSM083)

The complex plane (the Argand plane):

- examine and use addition of complex numbers as vector addition in the complex plane (ACMSM084)
- examine and use multiplication as a linear transformation in the complex plane (ACMSM085)
- identify subsets of the complex plane determined by relations, such as $|z - 3i| \le 4$, $\frac{\pi}{4} < Arg(z) \le \frac{3\pi}{4}$, $Re(z) > Im(z)$, and $|z - 1| = 2|z - i|$ (ACMSM086)

Roots of complex numbers:

- determine and examine the n^{th} roots of unity and their location on the unit circle (ACMSM087)
- determine and examine the n^{th} roots of complex numbers and their location in the complex plane (ACMSM088)

Factorisation of polynomials:

- prove and apply the factor theorem and the remainder theorem for polynomials (ACMSM089)
- consider conjugate roots for polynomials with real coefficients (ACMSM090)
- solve simple polynomial equations (ACMSM091)

Unit 3 – Topic 2: Functions and sketching graphs

Functions:

- determine when the composition of two functions is defined (ACMSM092)
- find the composition of two functions (ACMSM093)
- determine if a function is one-to-one (ACMSM094)
- consider inverses of one-to-one function (ACMSM095)
- examine the reflection property of the graph of a function and the graph of its inverse (ACMSM096)
- sketching graphs (ACMSM097)
- use and apply the notation $|x|$ for the absolute value for the real number x and the graph of $y = |x|$ (ACMSM098)
- examine the relationship between the graph of $y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$, $y = |f(x)|$ and $y = f(|x|)$ (ACMSM099)
- sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree (ACMSM100)

Unit 3 – Topic 3: Vectors in three dimensions

Vector and Cartesian equations:

- introduce Cartesian coordinates for three-dimensional space, including plotting points (ACMSM103)
- determine a 'corresponding' Cartesian equation in the two-dimensional case (ACMSM104)
- determine an equation of a straight line and straight-line segment, given the position of two points, or equivalent information, in both two and three dimensions (ACMSM105)
- determine Cartesian equations of a plane and of regions in a plane (ACMSM108)

Systems of linear equations:

- recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations (ACMSM109)
- examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables (ACMSM110)

Unit 4 – Topic 1: Integration and Applications of Integration

Integration techniques:

- integrate using the trigonometric identities $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos x = \frac{1}{2}(1 - \sin 2x)$ and $1 + \tan^2 x = \sec^2 x$ (ACMSM116)
- use substitution $u = g(x)$ to integrate expressions of the form $f(g(x))g'(x)$ (ACMSM117)
- establish and use the formula $\int \frac{1}{x} dx = \ln|x| + c$ for $x \neq 0$ (ACMSM118)
- find and use the inverse trigonometric functions: $\arcsin$e, arccosine and arctangent (ACMSM119)
- find and use the derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent (ACMSM120)
- integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$ (ACMSM121)
- use partial fractions where necessary for integration in simple cases (ACMSM122)
- integrate by parts (ACMSM123)

Applications of integral calculus:

- calculate areas between curves determined by functions (ACMSM124)
- determine volumes of solids of revolution about either axis (ACMSM125)
- use numerical integration using technology (ACMSM126)

Unit 4 – Topic 2: Rates of Change and Differential Equations

- use implicit differentiation to determine the gradient of curves whose equations are given in implicit form (ACMSM128)
- related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ (ACMSM129)
- solve simple first-order differential equations of the form $\frac{dy}{dx} = f(x)$, differential equations of the form $\frac{dy}{dx} = g(y)$ and, in general, differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables (ACMSM130)
- examine slope (direction or gradient) fields of a first order differential equation (ACMSM131)
- formulate differential equations including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved (ACMSM132)

Accreditation

The accreditation period for this course has been renewed from 1 January 2022 until 31 December 2025.

During the accreditation period required amendments can be considered via established processes.

Should outcomes of the Years 9-12 Review process find this course unsuitable for inclusion in the Tasmanian senior secondary curriculum, its accreditation may be cancelled. Any such cancellation would not occur during an academic year.

Version History

Version 1 – Accredited on 13 August 2017 for use from 1 January 2018. This course replaces MTS415114 Mathematics Specialised that expired on 31 December 2017.

Accreditation renewed on 22 November 2018 for the period 1 January 2019 until 31 December 2021.

Version 1.a - Renewal of Accreditation on 14 July 2021 for the period 31 December 2021 until 31 December 2025, without amendments.

Appendix 1

GLOSSARY

Addition of matrices (See Matrix)
If A and B are matrices with the same dimensions, and the entries of A are a_{ij} and the entries of B are b_{ij}, then the entries of $A + B$ are $a_{ij} + b_{ij}$.

For example, if $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix}$, then $A + B = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$.

Angle sum and difference identities
$\sin(A + B) = \sin A \cos B + \sin B \cos A$

$\sin(A - B) = \sin A \cos B - \sin B \cos A$

$\cos(A + B) = \cos A \cos B - \sin B \sin A$

$\cos(A - B) = \cos A \cos B + \sin B \sin A$

Arithmetic sequence
An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence $2, 5, 8, 11, 14, 17, \ldots$ is an arithmetic sequence with common difference 3.

If the initial term of an arithmetic sequence is a and the common difference of successive members is d, then the n^{th} term, t_n, of the sequence is given by $t_n = a + (n - 1)d$ where $n \geq 1$.

A recursive definition is $U_n = U_{n-1} + d$ where $n \geq 1$.

Arithmetic series
An arithmetic series is the sum of an arithmetic sequence $S_n = U_1 + U_2 + U_3 + \ldots + U_n = \sum_{r=1}^{n} U_r$.

The infinite series is given by $S_\infty = U_1 + U_2 + U_3 + \ldots = \sum_{r=1}^{\infty} U_r$. This can be found by evaluating $\lim_{n \to \infty} S_n$.

Argument (abbreviated Arg)
If a complex number is represented by a point P in the complex plane, then the argument of z, denoted $Arg(z)$, is the angle theta that OP makes with the positive real axis O_x, with the angle measured anticlockwise from O_x. The principal value of the argument is the one in the interval $(-\pi, \pi]$.

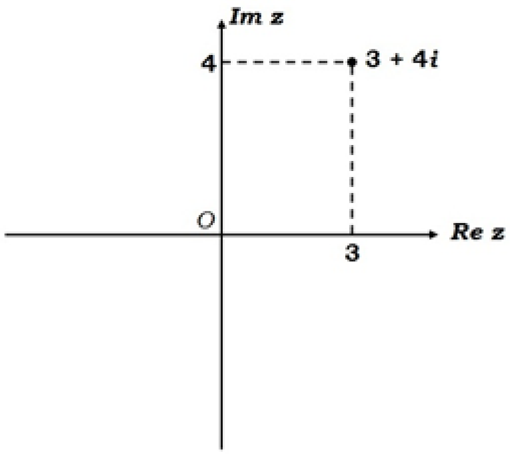
Complex arithmetic
If $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$, then:

- $z_1 + z_2 = \left(x_1 + x_2\right) + \left(y_1 + y_2\right)i$
- $z_1 - z_2 = \left(x_1 - x_2\right) + \left(y_1 - y_2\right)i$
- $z_1 \times z_2 = \left(x_1 x_2 - y_1 y_2\right) + \left(x_1 y_2 + x_2 y_1\right)i$
- $z_1 \times (0 + 0i) = 0$ *Note*: $0 + 0i$ is usually written as `**0**`
- $z_1 \times (1 + 0i) = z_1$ *Note*: $1 + 0i$ is usually written as 1

Complex conjugate
For any complex number $z = x + yi$, its conjugate is $\bar{z} = x - yi$. The following properties hold:

- $\overline{z_1 . z_2} = \overline{z_1} . \overline{z_2}$
- $\overline{\left(z_1/z_2\right)} = \overline{z_1}/\overline{z_2}$
- $\bar{z} = [z]^2$
- $z + \bar{z}$ is real

Complex plane (Argand plane)
The complex plane is a geometric representation of the complex numbers established by the real axis and the orthogonal imaginary axis. The complex plane is sometimes called the *Argand* plane.

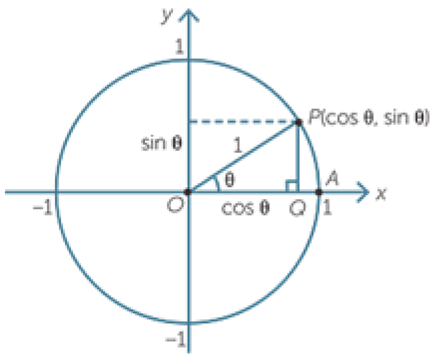


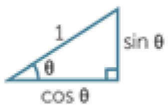
Convergent Sequences
A sequence $\{a_n\}$ converges to a finite number L if given $\varepsilon > 0$, however small, there exists N (dependent on ε) such that $|a_n - L| < \varepsilon$, provided that $n > N$.

$\lim_{n \to \infty} a_n = L.$

Cosine and Sine functions
Since each angle θ measured anticlockwise from the positive x-axis determines a point P on the unit circle, we will define:

- the cosine of θ to be the x-coordinate of the point P
- the sine of θ to be the y-coordinate of the point P
- the tangent of θ is the gradient of the line segment OP





De Moivre's Theorem
For all integers n, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Determinant of a 2 x 2 matrix
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant of A is denoted by $\det A = ad - bc$.

If $\det A \neq 0$,

- the matrix A has an inverse
- the simultaneous linear equations $ax + by = e$ and $cx + dy = f$ have a unique solution
- the linear transformation of the plane, defined by A, maps the unit square, $O(0, 0), B(0, 1), C(1, 1), D(1, 0)$, to a parallelogram $OB'C'D'$ of area $|\det A|$
- the sign of the determinant determines the orientation of the image of a figure under the transformation defined by the matrix.

Dimension (or Size) of a matrix
Two matrices are said to have the same dimensions (or size) if they have the same number of rows and columns.

For example, the matrices $\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ and $\begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$ have the same dimensions. They are both 2×3 matrices.

An $m \times n$ matrix has m rows and n columns.

Difference Method (or Method of Differences)
The method of differences can be used determine some 'special series', when we are not given the sum of the series.

For example, as $n^2 - (n-1)^2 = 2n - 1$, this will mean that $n^2 = 2\sum_{r=1}^{n} n^2 - 1$.

Divergent Sequences
A sequence $\{a_n\}$ diverges to ∞, if given $K > 0$, however great, there exists N (dependent on K) such that $a_n > K$, provided $n > N$.

$\lim_{n \to \infty} a_n = \infty$

Double angle formulae
$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Entries (Elements) of a matrix
The symbol a_{ij} represents the (i, j) entry which occurs in the i^{th} row and the j^{th} column.

For example, a general 3×2 matrix is $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$, and a_{32} is the entry in the third row and the second column.

Geometric sequence
A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the *common ratio*. For example, the sequence $6, 12, 24, \ldots$ is a geometric sequence with common ratio 2.

Similarly, the sequence $40, 20, 10, 5, 2.5, \ldots$ is a geometric sequence with common ratio $\frac{1}{2}$.

If the initial term of a geometric sequence is a, and the common ratio of successive members is r, then the n^{th} term, t_n, of the sequence is given by $t_n = ar^{n-1}$ for $n \geq 1$.

A recursive definition is $U_n = rU_{n-1}$ where $U_1 = a$.

Geometric series

A geometric series is the sum of a geometric sequence $S_n = U_1 + U_2 + U_3 + \ldots + U_n = \sum_{r=1}^{n} U_r$

$S_n = \dfrac{a\left(1 - r^n\right)}{(1 - r)}$ where $r \neq 1$.

The infinite series $S_\infty = U_1 + U_2 + U_3 + \ldots = \sum_{r=1}^{\infty} U_r$ can be found by evaluating $\lim_{n \to \infty} S_n$.

$S_\infty = \dfrac{a}{1 - r}$, provided $|r| < 1$.

Imaginary part of a complex number
A complex number z may be written as $x + yi$, where x and y are real, and then y is the imaginary part of z. It is denoted by $Im(z)$.

Implicit differentiation
When variables x and y satisfy a single equation, this may define y as a function of x even though there is no explicit formula for y in terms of x. *Implicit differentiation* consists of differentiating each term of the equation as it stands and making use of the chain rule. This can lead to a formula for $\frac{dy}{dx}$. For example, if $x^2 + xy^3 - 2x + 3y = 0$, then $2x + x\left(3y^2\right)\frac{dy}{dx} + y^3 - 2 + 3\frac{dy}{dx} = 0$, and so $\frac{dy}{dx} = \frac{2 - 2x - y^3}{3xy^2 + 3}$.

Inverse trigonometric functions
The inverse sine function, $y = \sin^{-1}x$

If the domain for the $\sin$e function is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, a one-to-one function is formed and so an inverse function exists.

The inverse of this restricted $\sin$e function is denoted by $\sin^{-1}$, and is defined by $\sin^{-1}: [-1, 1] \to R$, $\sin^{-1}x = y$ where$\sin y = x$, $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\sin^{-1}$ is also denoted by $\arcsin$.

The inverse cosine function, $y = \cos^{-1}x$
If the domain of the $\cos$ine function is restricted to $[0, \pi]$, a one-to-one function is formed and so the inverse function exists.

$\cos^{-1}x$, the inverse function of this restricted $\cos$ine function, is defined by $\cos^{-1}: [-1, 1] \to R$, $\cos^{-1}x = y$ where $\cos y = x$, $y \in [0, \pi]$.

$\cos^{-1}$ is also denoted by $\arccos$.

The inverse tangent function, $y = \tan^{-1}x$

If the domain of the $\tan$gent function is restricted to $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, a one-to-one function is formed and so the inverse function exists.

$\tan^{-1}: R \to R$, $\tan^{-1}x = y$, where $\tan y = x$, $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan^{-1}$ is also denoted by $\arctan$.

Leading diagonal
The leading diagonal of a square matrix is the diagonal which runs from the top left corner to the bottom right corner.

Linear Transformation Defined by a 2x2 matrix
The matrix multiplication $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$ defines a transformation $T(x, y) = (ax + by, cx + dy)$.

Linear Transformations in 2-dimensions
A linear transformation in the plane is a mapping of the form $T(x, y) = (ax + by, cx + dy)$.

A transformation T is linear if and only if $T\left(\alpha\left(x_1, y_1\right) + \beta\left(x_2, y_2\right)\right) = \alpha T\left(x_1, y_1\right) + \beta T\left(x_2, y_2\right)$.

Linear transformations include:

- rotations around the origin
- reflections in lines through the origin
- dilations.

Translations are not linear transformations.

MacLaurin's Series
A Maclaurin series is a special power series expansion for $f(x)$.

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^3(0)}{3!}x^3 + \ldots + \frac{f^n(0)}{n!}x^n + \ldots$$

It equals $f(x)$ whenever the series converges. MacLaurin series converge for all real x, some for a subset of x and others for no values of x.

Matrix (matrices)

A matrix is a rectangular array of elements or entries displayed in rows and columns. For example, $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ are both matrices.

Matrix A is said to be a 3×2 matrix (three rows and two columns), while B is said to be a 2×3 matrix (two rows and three columns).

A *square matrix* has the same number of rows and columns.

A *column matrix* (or *vector*) has only one column.

A *row matrix* (or *vector*) has only one row.

Matrix algebra of 2×2 matrices
If A, B and C are 2×2 matrices, I the 2×2 (*multiplicative*) *identity matrix*, and 0 the 2×2 *zero matrix*, then:

$A + B = B + A$ (commutative law for addition)
$(A + B) + C = A + (B + C)$ (associative law for addition)
$A + 0 = A$ (additive identity)
$A + (-A) = 0$ (additive inverse)
$(AB)C = A(BC)$ (associative law for multiplication)
$AI = A = IA$ (multiplicative identity)
$A(B + C) = AB + AC$ (left distributive law)
$(B + C)A = BA + CA$ (right distributive law)

Matrix multiplication
Matrix multiplication is the process of multiplying a matrix by another matrix. The product AB of two matrices A and B with *dimensions* $m \times n$ and $p \times q$ is defined if $n = p$. If it is defined, the product AB is an $m \times q$ matrix and it is computed as shown in the following example.

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} 6 & 10 \\ 11 & 3 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 94 & 34 \\ 151 & 63 \end{bmatrix}$$

The entries are computed as shown:

$1 \times 6 + 8 \times 11 + 0 \times 12 = 94$
$1 \times 10 + 8 \times 3 + 0 \times 4 = 34$
$2 \times 6 + 5 \times 11 + 7 \times 12 = 151$
$2 \times 10 + 5 \times 3 + 7 \times 4 = 63$

The entry in row i and column j of the product AB is computed by 'multiplying' row i of A by column j of B as shown.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$, then

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$$

Modulus (Absolute value) of a complex number
If z is a complex number and $z = x + yi$, then the modulus of z is the distance of z from the origin in the Argand plane. The modulus of z is denoted by $|z| = \sqrt{x^2 + y^2}$.

Multiplication by a scalar
Let a be a non-zero vector and k a positive real number (scalar), then the *scalar multiple* of a by k is the vector ka which has magnitude $|k||a|$ and the same direction as a. If k is a negative real number, then ka has magnitude $|k||a|$ but is directed in the opposite direction to a.

Some properties of scalar multiplication are:

$$k(a + b) = ka + kb$$
$$h(k(a)) = (hk)a$$
$$Ia = a$$

(Multiplicative) identity matrix
A (*multiplicative*) *identity matrix* is a square matrix in which all the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter I.

For example, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are both identity matrices.

There is an identity matrix for each order of square matrix. When clarity is needed, the order is written with subscript, I_n.

Multiplicative inverse of a square matrix
The inverse of a square matrix A is written as A^{-1} and has the property that $AA^{-1} = I$.

Not all square matrices have an inverse. A matrix that has an inverse is said to be *invertible*.

Multiplicative inverse of a 2×2 matrix:
The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, when $\det A \neq 0$.

Scalar multiplication (Matrices)

Scalar multiplication is the process of multiplying a matrix by a scalar (number). For example, forming the product $10\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$ is an example of the process of scalar multiplication.

In general, for the matrix A with entries a_{ij}, the entries of kA are ka_{ij}.

Polar form of a complex number
For a complex number z, let $\theta = arg(z)$. Then $z = r(\cos\theta + i\sin\theta)$ is the polar form of z.

Power Series
A *power series* is an *infinite series* $a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$ where $a_0, a_1, a_2, a_3, \ldots$ are real constants.

Principle of mathematical induction
Let there be associated with each positive integer n, a proposition $P(n)$.

If $P(1)$ is true, and for all k, $P(k)$ is true implies $P(k + 1)$ is true, then $P(n)$ is true for all positive integers n.

Products as sums and differences

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$
$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$
$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$
$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

Pythagorean identities

$\cos^2 A + \sin^2 A = 1$
$\tan^2 A + 1 = \sec^2 A$
$\cot^2 A + 1 = \mathrm{cosec}^2 A$

Rational function

A rational function is a function such that $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials. Usually, $g(x)$ and $h(x)$ are chosen so as to have no common factor of degree greater than or equal to 1, and the domain of f is usually taken to be $\{x \in R : h(x) \neq 0\}$.

Real part of a complex number
A complex number z may be written as $x + yi$, where x and y are real, and then x is the real part of z. It is denoted by $Re(z)$.

Reciprocal trigonometric functions

$\sec A = \frac{1}{\cos A}, \cos A \neq 0$
$\mathrm{cosec} A = \frac{1}{\sin A}, \sin A \neq 0$
$\cot A = \frac{\cos A}{\sin A}, \sin A \neq 0$

Root of unity

Given a complex number z such that $z^n = 1$, then the n^{th} roots of unity are: $\cos\left(\frac{2k\pi}{n}\right) + i\sin\left(\frac{2k\pi}{n}\right)$, where $k = 0, 1, 2, ..., n - 1$.

The points in the complex plane representing roots of unity lie on the unit circle.

The cube roots of unity are $z_1 = 1$, $z_2 = \frac{1}{2}\left(-1 + i\sqrt{3}\right)$, $z_3 = \frac{1}{2}\left(-1 - i\sqrt{3}\right)$.

Note: $z_3 = \overline{z_2}$; $z_3 = \frac{1}{z_2}$; and $z_2 z_3 = 1$.

Separation of variables
Differential equations of the form $\frac{dy}{dx} = g(x)h(y)$ can be rearranged as long as $h(y) \neq 0$ to obtain $\frac{1}{h(y)}\frac{dy}{dx} = g(x)$.

Sigma Notation Rules

$\sum_{r=1}^{n} f(r) = f(1) + f(2) + f(3) + ... + f(n)$
$\sum_{r=1}^{n} (f(r) + g(r)) = \sum_{r=1}^{n} f(r) + \sum_{r=1}^{n} g(r)$
$\sum_{r=1}^{n} kf(r) = k\sum_{r=1}^{n} f(r)$

Singular matrix
A matrix is singular if $\det A = 0$. A singular matrix does not have a multiplicative inverse.

Vector equation of a plane
Let a be a position vector of a point A in the plane, and n a normal vector to the plane, then the plane consists of all points P whose position vector p satisfies $(p - a)n = 0$. This equation may also be written as $pn = an$, a constant.

(If the normal vector n is the vector (l, m, n) in ordered triple notation and the scalar product $an = k$, this gives the Cartesian equation $lx + my + nz = k$ for the plane.)

Vector equation of a straight line
Let a be the position vector of a point on a line l, and u any vector with direction along the line. The line consists of all points P whose position vector p is given by $p = a + tu$ for some real number t.

(Given the position vectors of two points on the plane a and b, the equation can be written as $p = a + t(b - a)$ for some real number t.)

Zero matrix

A zero matrix is a matrix if all of its entries are zero. For example, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are zero matrices.

There is a zero matrix for each *size* of matrix.

Appendix 2

LINE OF SIGHT – Mathematics Specialised Level 4

Learning Outcomes	Separating Out Content From Skills in 4 of the Learning Outcomes		Criteria and Elements	Content
utilise concepts and techniques from sequences and series, complex numbers, matrices and linear algebra, function and equation study and calculus	Sequences and Series	concepts and techniques; problem solving; reasoning skills; interpret and evaluate ; select and use appropriate tools	C4 E1-8 C4 E1-8; C2 E1,4,7 C2 E1,3,4,7 C1 E5 C1 E2,5,6; C2 E5-6	Sequences and Series (can include some, non examinable applications)
solve problems using concepts and techniques drawn from algebraic processes, sequences and series, complex numbers, matrices and linear algebra, function and equation study and calculus	Matrices and Linear Algebra	concepts and techniques; problem solving; reasoning skills; interpret and evaluate; select and use appropriate tools	C5 E1-8 C5 E1-8; C2 E1,4,7 C2 E1,3,4,7 C3 E1; C1 E5 C1 E2,5,6; C2 E5-6	Matrices and Linear Algebra (can include some, non examinable applications)
apply reasoning skills in the contexts of algebraic processes, sequences sand series, complex numbers, matrices and linear algebra, function and equation study and calculus	Calculus – Part 1: implicit differentiation, fundamental theorem,definite integrals	concepts and techniques; problem solving; reasoning skills; interpret and evaluate; select and use appropriate tools	C6 E1-5 C6 E1-5; C2 E1,4,7 C2 E1,3,47 C1 E5 C1 E2,5,6; C2 E5-6	Calculus – Part 1 (can include some, non examinable applications)
interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems	Calculus – Part 2: techniques of integration, first order differential equations, applications	concepts and techniques; problem solving; reasoning skills; interpret and evaluate; select and use appropriate tools	C7 E1-6 C7 E1-6; C2 E1,4,7 C2 E1,3,47 C3 E1; C1 5 C1 E2,5,6; C2 E5-6	Calculus – Part 2 (can include some, non examinable applications)
	Complex Numbers	concepts and techniques; problem solving; reasoning skills; interpret and evaluate; select and use appropriate tools	C8 E1-6 C8 E1-6; C2 E1,4,7 C2 E1,3,4,7 C1 E5 C1 E2,5,6; C2 E5-6	Complex Numbers (can include some, non examinable applications)
communicate their arguments and strategies when solving problems			C1 E1-7	All content areas
plan activities and monitor and evaluate progress; use strategies to organise and complete activities, and meet deadlines in context of mathematics			C1 E1,2,5,6 C3 E5-7	All content areas
select and use appropriate tools, including computer technology, when solving			C1 E5-7 C2 E5-7	All content areas

mathematical problems				

Supporting documents including external assessment material

- MTS415118 TASC Exam Paper 2018.pdf (2018-11-22 12:07pm AEDT)
- MTS415118 - Assessment Panel Report 2018.pdf (2019-02-19 02:34pm AEDT)
- MTS415118 Mathematics Specialised TASC Exam Paper 2019.pdf (2019-11-19 06:10pm AEDT)
- MTS415118 Assessment Report 2019.pdf (2020-02-07 10:34am AEDT)
- MTS415118 Maths Specialised TASC Exam Paper 2020.pdf (2020-11-11 06:52pm AEDT)
- MTS415118 Assessment Report 2020.pdf (2021-01-20 04:08pm AEDT)
- MTS415118 Information Sheet.pdf (2021-04-19 12:20pm AEST)
- MTS415118 Mathematics Specialised TASC Exam Paper 2021.pdf (2021-11-17 10:02am AEDT)
- MTS415118 Assessment Report 2021.pdf (2022-01-31 12:43pm AEDT)
- MTS415118 Mathematics Specialised External Assessment Specifications.pdf (2023-03-27 11:19am AEDT)
- MTS415118 Mathematics Specialised TASC Exam Paper 2022.pdf (2022-11-15 08:38am AEDT)