DEPARTMENT OF EDUCATION

learners first

Discipline-based study

Mathematics

General Mathematics 3

Course document

independent schools tasmania

Catholic Education Tasmania

Years 9-12 Learning

Tasmanian Government

Table of contents

General Mathematics, 150 hours – Level 3

Focus area – Discipline-based study

Courses aligned to the Years 9 to 12 Curriculum Framework belong to one of the five focus areas of Discipline-based study, Transdisciplinary projects, Professional studies, Work-based learning and Personal futures.

General Mathematics Level 3 is a Discipline-based study course.

Discipline-based study includes content, core concepts and big ideas; enabling deep knowledge and understanding of the content and the application of what is learned. Learners consider accepted key disciplinary knowledge, apply distinctive ways of thinking and become increasingly independent learners. They use methodologies specific to the discipline to explore and strengthen their understanding of key concepts and develop deep knowledge, skills and understanding.

Discipline-based study courses have three key features that guide teaching and learning:

- specialist knowledge
- theories and concepts
- methodology and terminology.

Figure 1: Discipline-based Study diagram (developed by Years 9-12 Learning)

In this course learners will engage with specialist knowledge, core concepts and big ideas in the strands of algebra, finance, trigonometry, statistics and networks. Students will apply their knowledge and understanding through strategic selection and application of methodologies including problem solving, mathematical modelling and statistical investigations with and without the aid of technology.

Throughout the course, learners will demonstrate conceptual understanding through their fluency of calculation, mathematical reasoning and communication of mathematical ideas and information using appropriate conventions, terminology and representations.

Rationale

The *General Mathematics* Level 3 course is designed to develop learners' understanding of concepts and techniques drawn from number, including finance and algebra, as well as sequences, networks and decision mathematics, and statistics. This breadth of mathematical experience will enable learners to apply mathematical concepts and perform techniques to solve applied problems, synthesise mathematical information, and design and conduct mathematical investigations to calculate and communicate possible solutions.

The *General Mathematics* Level 3 course will enable learners to develop the foundations for study in many disciplines at tertiary level and engage in applications of those disciplines. Mathematics and numeracy provide a way of interpreting everyday practical situations and provide the basis for many informed personal decisions.

This course will enable learners to develop their mathematical expertise such that they may contribute productively to an ever-changing global economy, with rapid revolutions in technology and global and local social challenges. This is a key factor in ensuring Tasmania and Australia's current and emerging needs are met, as an economy competing globally requires substantial numbers of professionals with a strong grounding in mathematics. This course is designed to be supportive of learners pursuing both STEM and non-STEM specific pathways and professions including teaching, social sciences, allied health, accounting, business and marketing.

The purpose of [Years 9 to 12 Education] is to enable all learners to achieve their potential through Years 9 to 12 and beyond in further study, training or employment.

Years 9 to 12 Education enables personal empowerment, cultural transmission, preparation for citizenship and preparation for work.

This course is built on the principles of access, agency, excellence, balance, support and achievement as part of a range of programs that enables learners to access a diverse and flexible range of learning opportunities suited to their level of readiness, interests and aspirations.

Learning outcomes

On successful completion of this course, learners will be able to:

1. communicate arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
2. plan, organise and manage learning in order to complete tasks and evaluate progress
3. apply modelling, problem solving and mathematical reasoning to interpret, represent and justify the reasonableness of solutions to problems and answers to statistical questions
4. choose and use technology appropriately and effectively
5. understand and apply concepts and techniques of bivariate data analysis and time series analysis using the statistical investigation process
6. understand and apply concepts and techniques to model and solve problems involving growth and decay in sequences
7. understand and apply concepts and techniques to solve problems involving loans, investment and annuities
8. understand and apply concepts and techniques to represent, analyse and solve problems in the two-dimensional plane.

Integration of general capabilities and cross-curriculum priorities

The general capabilities addressed specifically in this course are:

- Critical and creative thinking
- Information and communication technology capability
- Literacy
- Numeracy
- Personal and social capability

The cross-curriculum priorities enabled through this course are:

- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia's engagement with Asia
- Sustainability

Course description

General Mathematics Level 3 enables learners to extend their mathematical experience beyond Year 10 with increasing sophistication. It provides increasingly abstract scenarios for incorporating mathematical arguments and problem solving in situations involving growth and decay, standard financial models, bivariate data analysis, time series analysis, trigonometry, geometry, networks and decision mathematics.

Learners will apply mathematical concepts and techniques to communicate reasoned arguments, solve problems and explain reasonableness of solutions.

In this course, learners will model and investigate situations with and without the use of technology. By working collaboratively, they will reflect upon and extend their own thinking.

Pathways

- *General Mathematics* Level 3 has a clear pathway from Australian Curriculum Mathematics F-10 and the proposed *General Mathematics* Level 2.
- *General Mathematics* Level 3 provides a pathway into a wide range of educational and employment opportunities, including continuing their studies at university or TAFE. While the successful completion of this course will gain entry into some post-secondary courses, other courses may require the successful completion of *Mathematics Methods* – Level 4.

Course requirements

Access

This course requires learners to collaborate with others.

Previously submitted work cannot be used in meeting the requirements of *General Mathematics* Level 3. Therefore, a learner cannot use work including, but not limited to, an independent study, folio, project or assignment that has already been presented for assessment for a previously or concurrently studied TASC accredited or recognised senior secondary course.

Resource requirements

Learners are required to have access to graphics calculators that meets the requirements as outlined in the External Assessment Specifications.

Computers and the internet are required to enable learners' access to information and data sources. Additionally, the use of computer software packages is strongly recommended as an aid to student learning and mathematical development. In particular, digital spreadsheets should be available.

Course structure and delivery

Structure

This course consists of three 50-hour modules.

Module 1: Mathematical modelling, problem solving and the statistical investigation process

Module 2: Statistical analysis and situations involving growth and decay in sequences

Module 3: Loans, investment and annuities, and practical problems in the two-dimensional plane

Delivery

Module 1 should be delivered concurrently with modules 2 and 3 which can be taught in any order.

Course content

Module 1: Mathematical modelling, problem solving and the statistical Investigation process

Within this module, students will apply mathematical processes in complex contexts with open-ended aspects that require problem-solving, modelling or investigative techniques or approaches. Students will apply, analyse and discuss these applications and their results. Additionally, students will apply computational thinking and use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, calculate results and analyse the situations described above.

Module 1 learning outcomes

The following learning outcomes are a focus for this module:

1. communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
2. plan, organise and manage learning in order to complete tasks and evaluate progress
3. apply modelling, problem solving and mathematical reasoning to interpret, represent and justify the reasonableness of solutions to problems and answers to statistical questions
4. choose and use technology appropriately and effectively.

Module 1 content

Key knowledge and skills

Apply mathematical processes in contexts that require problem-solving, modelling or investigation

- formulate the concepts, technique and models required to solve and interpret mathematical problems
- identify, select and apply facts, concepts, models and techniques needed to investigate and analyse statistical features of a data set with several variables that may include time series data
- select and implement models to investigate patterns of growth and decay in discrete situations and analyse to solve an extended problem
- select and apply standard financial models to investigate and analyse a financial situation that requires the use of increasingly sophisticated models to complete the analysis
- select and apply the mathematical concepts, models and techniques needed to represent, analyse and solve an extended problem in the two-dimensional plane
- interpret and report the results of statistical investigations and mathematical modelling or problem-solving tasks in terms of the context under consideration, including:

- assessing the reasonableness of results
- discussing any assumptions in application of these models and any limitations of the model
- drawing conclusions in light of the results obtained.

Use technology and other sources to develop ideas and find solutions

- access and manage information from digital and non-digital sources to develop mathematical ideas
- validate information taken from digital and non-digital sources through secondary sources or experimentation
- use technology to carry out numerical, graphical and symbolic computation as applicable
- select an appropriate functionality of technology in a variety of mathematical contexts and provide a rationale for these selections
- use appropriate domain and range specifications to illustrate key features of graphs
- apply suitable constraints and conditions, as applicable, to carry out required computations
- identify the relationship between numerical, graphical and symbolic forms of information about models and equations and the corresponding features of those models and equations
- distinguish between exact and approximate presentations of mathematical results produced by technology, and interpret these results to a specified degree of accuracy in terms of a given number of decimal places or significant figures
- produce tables of values, families of graphs and collections of other results using technology, which support general analysis in problem-solving, investigative and modelling contexts
- specify the similarities and differences between formal mathematical expressions and their representation by technology
- relate the results from a particular technology application to the nature of a particular mathematical task, investigative, problem solving or modelling, and verify these results
- specify the process used to develop a solution to a problem using technology and communicate the key stages of mathematical reasoning, formulation, solution and interpretation used in this process.

Module 1 work requirements

This module includes the following work requirements:

- one statistical investigation
- one extended response involving mathematical modelling or problem solving.

See Appendix 3 for the full specifications of the work requirements of this course.

Module 1 assessment

This module has a focus on criteria 1, 2, 3 and 4.

Module 2: Statistical analysis and situations involving growth and decay in sequences

This module contains two topics:

- statistical analysis
- growth and decay in sequences.

‘Statistical analysis’ is to be taught within the framework of the statistical investigation process. The first sub-topic ‘bivariate data analysis’ introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a tool for modelling and analysing linear associations. The second subtopic ‘Time series analysis’ furthers students’ study of statistics by introducing them to the concepts and techniques of time series analysis, including the use of simple moving averages, seasonal indices and fitting least-squares lines to model and describe long term trends in time series data.

‘Growth and decay in sequences’ employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population, or the decrease in the value of a car over time.

Module 2 learning outcomes

The following learning outcomes are a focus for this module:

1. communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
2. plan, organise and manage learning in order to complete tasks and evaluate progress
5. understand and apply concepts and techniques of bivariate data analysis and time series analysis using the statistical investigation process
6. understand and apply concepts and techniques to model and solve problems involving growth and decay in sequences.

Module 2 content

Topic 1 – statistical analysis

Subtopics:

- bivariate data analysis
- time series analysis.

Key knowledge and skills

Bivariate data analysis

Identifying and describing associations between variables:

- review the statistical investigation process; for example, identify a problem and pose a statistical question, collect or obtain data, analyse the data, interpret and communicate the results
- construct two-way frequency tables and determine the associated row and column sums and percentages
- use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association
- describe an association in terms of differences observed in percentages across categories in a systematic and concise manner and interpret this in the context of the data
- construct a scatterplot to identify patterns in the data suggesting the presence of an association
- describe an association between two numerical variables in terms of direction, positive or negative, form, linear or non-linear, and strength, strong, moderate or weak
- calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association.

Modelling and analysing linear relationships:

- review straight line equations and graphs ($y = mx + c$) by constructing graphs from given equations and solving equations from given graphs
- identify the response, dependent variable and the explanatory, independent, variable
- use a scatterplot to identify the nature of the relationship between variables
- model a linear relationship by fitting a least-squares line to the data with and without the aid of technology
- use a residual plot to assess the appropriateness of fitting a linear model to the data
- interpret, in context, the intercept and gradient of the fitted line used to model and analyse a practical situation
- use the coefficient of determination (r^2) to assess the strength of a linear association in terms of the explained variation
- use the equation of a fitted line to make predictions
- distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation
- communicate the results of bivariate data analysis in a systematic and concise manner.

Recognising association and causation:

- recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them
- identify possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable, and communicate these explanations in a systematic and concise manner.

Implementing the data investigation process:

- implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or between two numerical variables; for example, is there an association between attitude to capital punishment, agree with, no opinion, disagree with, and sex, male, female? is there an association between height and foot length?

Time series analysis

Describing and interpreting patterns in time series data:

- construct time series plots
- describe time series plots by identifying features such as trend, long term direction, seasonality, systematic, calendar-related movements, and irregular fluctuations, unsystematic, short-term fluctuations, and recognise when there are outliers; for example, one-off unanticipated events.

Analysing time series data:

- smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process
- calculate seasonal indices by using the average percentage method
- de-seasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process
- fit a least-squares line to model long-term trends in time series data.

The data investigation process:

- implement the statistical investigation process to answer questions that involve the analysis of time series data.

Topic 2 - growth and decay in sequences

Subtopics:

- the arithmetic sequence
- the geometric sequence
- sequences generated by first-order linear recurrence relations.

Key knowledge and skills

The arithmetic sequence

- use recursion to generate an arithmetic sequence
- represent terms of an arithmetic sequence in both tabular and graphical form
- use arithmetic sequences to model linear growth and decay in discrete situations
- deduce a rule for the nth term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, $t_n = a + (n-1)d$ and use this rule to make predictions
- use arithmetic sequences to model and analyse practical situations involving linear growth or decay
- determine the sum, to n terms, of an arithmetic sequence, represented as $S_n = \frac{n}{2}(a + l)$ or $S_n = \frac{n}{2}(2a + (n-1)d)$.

The geometric sequence

- use recursion to generate a geometric sequence
- represent the terms of a geometric sequence in both tabular and graphical form
- use geometric sequences to model exponential growth and decay in discrete situations
- deduce a rule for the nth term of a particular geometric sequence from the pattern of the terms in the sequence, $t_n = ar^{n-1}$ and use this rule to make predictions
- use geometric sequences to model and analyse, numerically, or graphically only, practical problems involving geometric growth and decay
- determine the sum, to n terms, of an arithmetic sequence, represented as $S_n = \frac{a(1-r^n)}{1-r}$, where $r \neq 1$
- determine the sum of an infinite geometric series, represented as $S\infty = a_1 / (1 - r)$.

Sequences generated by first-order linear recurrence relations

- use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form
- recognise that a sequence generated by a first-order linear recurrence relation can have a long-term increasing, decreasing or steady-state solution
- use first-order linear recurrence relations to model and analyse, numerically or graphically only, practical problems involving growth or decay, with or without the aid of technology.

Module 2 work requirements

This module includes the following work requirement:

- one connected series of short responses applying concepts and techniques to solve problems.

See Appendix 3 for the full specifications of the work requirements of this course.

Module 2 assessment

This module has a focus on criteria 1, 2, 5 and 6.

Module 3: Loans, investment and annuities, and practical problems in the two-dimensional plane

This module contains one core topic and two elective topics of which one must be chosen:

Core:

- investment, loans and annuities.

Elective options:

- applications of trigonometry and Earth geometry
- networks and decision mathematics.

'Investment, loans and annuities' aims to provide students with sufficient knowledge and understanding of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage, depreciation on plant and equipment, contributions to superannuation and making investments. Study of this topic will assist students to develop awareness of mechanisms to optimise their financial position, both now and into the future, justifying their thinking and reasoning mathematically.

'Trigonometry and Earth geometry' extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation. Additionally, it enables students to solve problems relating to identifying locations and measuring distances between locations on the Earth's surface and to make connections between longitudinal location and time zones which has practical implications for the global nature of the world of work, specifically relating to implications upon travel and connectivity.

'Networks and decision mathematics' builds students' capacity to graphically represent and model situations as an approach to decision-making. Knowledge of networks enables development of a logical sequence of tasks or a clear understanding of connections between people or items and project planning and management tools such as critical path analysis and the 'maximum-flow, minimum-cut' theorem. Study of this topic is important in developing students' ability to interpret a set of connections or sequence of tasks as a concise diagram in order to solve related problems and to use critical path analysis in the optimisation of real-life problems.

Module 3 learning outcomes

The following learning outcomes are a focus for this module:

1. communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
2. plan, organise and manage learning in order to complete tasks and evaluate progress
7. understand and apply concepts and techniques to solve problems involving loans, investment and annuities
8. understand and apply concepts and techniques to represent, analyse and solve problems in the two-dimensional plane.

Module 3 content

Topic 1 – investment, loans and annuities

Subtopics:

- compound interest investments and loans
- reducing balance loans
- annuities and perpetuities.

Key knowledge and skill

Compound interest investments and loans

- use a recurrence relation to model a compound interest loan or investment and investigate, numerically or graphically, the effect of the interest rate and the number of compounding periods on the future value of the loan or investment
- calculate the effective annual rate of interest, $E = (1+i)^n - 1$, and use the results to compare investment returns and cost of loans when interest is paid or charged daily, fortnightly, monthly, quarterly or six-monthly
- solve, with and without the aid of a calculator or computer-based financial software, problems involving compound interest loans or investments; for example:
  - calculate the future value (FV) or present value (PV) and the interest rate (i) of a compound interest investment using the formula $FV = PV(1+i)^n$
  - determine the number of compounding periods for an investment to exceed a given value
  - investigate the effect of varying the interest rate, the term or the compounding period on the future value of an investment, using technology.
- compare and contrast different investment strategies, performing appropriate calculations when needed
- solve practical problems, algebraically or graphically, involving compounding; for example, determine the impact of inflation on prices and wages, or depreciation tables on assets.

Reducing balance loans

- use a recurrence relation, algebraically, to model a reducing balance loan and investigate, numerically or graphically, the effect of the interest rate and repayment amount on the time taken to repay the loan
- calculate the depreciation of an asset using the declining-balance method using the formula $S = P(1-i)^n$, where S is the salvage value of the asset after n periods, P is the initial value of the asset, i is the depreciation rate per period expressed as a decimal, and n is the number of periods, as an application of the compound interest formula
- with and without the aid of a financial calculator or computer-based financial software, solve practical problems involving reducing balance loans; for example, determining the total loan amount and monthly repayments
- recognise credit cards as an example of a reducing balance loan and solve practical problems relating to credit cards; for example:
  - identify the various fees and charges associated with credit card usage
  - compare credit card interest rates with interest rates for other loan types
  - interpret credit card statements, recognising the implications of only making the minimum payment
  - understand what is meant by an interest-free period
  - calculate the compounding interest charged on a retail purchase, transaction or the outstanding balance for a given number of days, with or without the aid of technology.

Annuities and perpetuities

- identify an annuity as an investment account with regular, equal contributions and interest compounding at the end of each period, or as a single sum investment from which regular, equal withdrawals are made

- use a recurrence relation, algebraically, to model an annuity and investigate, numerically or graphically, the effect of varying the amount invested, the frequency of each contribution, the interest rate or the payment amount on the duration or future value of the annuity
- use a table of interest factors to perform annuity calculations; for example, calculating the present or future value of an annuity, the contribution amount required to achieve a given future value or the single sum that would produce the same future value as a given annuity
- with the aid of technology, solve problems involving annuities, including perpetuities, $P = \frac{R}{i}$ as a special case, where i = effective interest rate.

Topic 2a - trigonometry and earth geometry

Subtopics:

- applications of trigonometry
- earth geometry and time zones.

Key knowledge and skills

Applications of trigonometry

- review the use of Pythagoras' theorem to solve problems involving right-angled triangles
- review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle
- determine the area of a triangle given two sides and an included angle by using the rule $Area = \frac{1}{2}ab \cdot sin(C)$, or given three sides by using Heron's rule, and solve related practical problems
- solve problems involving non-right-angled triangles using the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, ambiguous case excluded, and the cosine rule $c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$
- solve practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression, and the use of bearings in navigation.

Earth geometry and time zones

- understand and perform calculations in relation to great circles, small circles, latitude, longitude and angular distance
- find distances between two places on Earth on the same longitude
- use arc length and plane geometry to calculate distances in kilometres, along great and small circles associated with parallels of latitude and meridians of longitude
- locate positions on Earth's surface given latitude and longitude using GPS, a globe, an atlas and digital technologies
- calculate great circle distances between two points B and C using the spherical cosine rule $\cos\theta = \sin latP \cdot \sin latQ + \cos latP \cdot \cos latQ \cdot \cos \Delta long$, where θ is the angle subtended at the centre of the great circle by the great circle arc between two points P and Q and check using appropriate technology. Latitudes should be considered as positive above the equator and negative below the equator
- understand the link between longitude and time
- solve problems involving time zones in Australia and in neighbouring nations, making any necessary allowances for daylight saving
- solve problems involving co-ordinated universal time (UTC) and the international date line
- find time differences between two places on Earth

- solve problems associated with time zones; for example, internet and phone usage
- solve problems relating to travelling east and west, incorporating time zone changes
- carry out time and distance calculations involving world travel problems, including scenarios involving more than one destination with 'stop-overs'.

Topic 2b - graphs, networks and decision mathematics

Subtopics:

- graphs, paths and cycles
- trees and minimum connector problems
- critical path analysis
- flow networks and assignment problems.

Key knowledge and skills

Graphs, paths and cycles

- explain the meanings of the terms: graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph, digraph, arc, weighted graph and network
- identify practical situations that can be represented by a network, and construct such networks; for example, trails connecting camp sites in a national park, a social network, a transport network with one-way streets, a food web or the results of a round-robin sporting competition
- construct an adjacency matrix from a given graph or digraph
- explain the meaning of the terms: planar graph and face
- apply Euler's formula, $V + F - E = 2$ to solve problems relating to planar graphs
- explain the meaning of the terms: walk, trail, path, closed walk, closed trail, cycle, connected graph and bridge
- investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph by trial-and-error methods only
- explain the meaning of the terms: Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems; for example, the Königsberg Bridge problem or planning a garbage bin collection route
- explain the meaning of the terms: Hamiltonian graph and semi-Hamiltonian graph and use these concepts to investigate and solve practical problems; for example, planning a sight-seeing tourist route around a city, the travelling-salesman problem, by trial-and-error methods only.

Trees and minimum connector problems

- explain the meaning of the terms: tree and spanning tree and identify practical examples
- identify a minimum spanning tree in a weighted connected graph either by inspection or by using Prim's algorithm
- use minimal spanning trees to solve minimal connector problems; for example, minimising the length of cable needed to provide power from a single power station to substations in several towns.

Critical path analysis

- construct a network to represent the durations and interdependencies of activities that must be completed during the project; for example, preparing a meal

- use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project
- use ESTs and LSTs to locate the critical path or paths for the project
- use the critical path to determine the minimum time for a project to be completed
- calculate float times for non-critical activities.

Flow networks and assignment problems:

- solve small-scale network flow problems including the use of the 'maximum-flow minimum-cut' theorem; for example, determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank, the source, to a terminal, the sink
- use a bipartite graph or its tabular or matrix form to represent an assignment problem; for example, assigning four swimmers to the four places in a medley relay team to maximise the team's chances of winning
- determine the optimum assignment or assignments, by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems.

Module 3 work requirements

This module includes the following work requirement:

- one connected series of short responses applying concepts and techniques to solve problems.

See Appendix 3 for the full specifications of the work requirements of this course.

Module 3 assessment

This module has a focus on criteria 1, 2, 7 and 8.

Assessment

Criterion-based assessment is a form of outcomes assessment that identifies the extent of learner achievement at an appropriate endpoint of study. Although assessment as part of the learning program is continuous, much of it is formative and is done to help learners identify what they need to do to attain the maximum benefit from their study of the course. Therefore, assessment for summative reporting to TASC will focus on what both teacher and learner understand to reflect endpoint achievement.

The standard of achievement each learner attains on each criterion is recorded as a rating 'A', 'B' or 'C', according to the outcomes specified in the standards section of the course.

A 't' notation must be used where a learner demonstrates any achievement against a criterion less than the standard specified for the 'C' rating.

A 'z' notation is to be used where a learner provides no evidence of achievement at all.

Internal assessment of all criteria will be made by the provider. Providers will report the learner's rating for each criterion to TASC.

TASC will supervise the external elements of designated criteria which will be indicated by an asterisk (*). The ratings obtained from the external assessments will be used in addition to internal ratings from the provider to determine the final award.

Criteria

The assessment for *General Mathematics* Level 3 will be based on the degree to which the learner can:

1. communicate mathematical ideas and information and apply mathematical conventions
2. manage and take responsibility for learning and evaluate mathematical development
3. apply mathematical and statistical models to investigate, represent and analyse real-world situations and solve problems*
4. use digital technology and other sources to develop mathematical ideas and find solutions to mathematical problems
5. interpret concepts and apply mathematical techniques to solve problems involving bivariate data analysis and time series analysis using the statistical investigation process*
6. interpret concepts and apply mathematical techniques to model patterns and solve problems involving growth and decay in sequences*
7. interpret concepts and apply mathematical techniques to solve problems involving standard financial models*
8. interpret concepts and apply mathematical techniques to represent, analyse and solve practical problems in the two-dimensional plane.*

*denotes criteria that are both internally and externally assessed.

	Module 1	Module 2	Module 3
Criteria focus	1,2,3,4	1,2,5,6	1,2,7,8

Standards

Criterion 1: communicate complex mathematical ideas and information and apply mathematical conventions

This criterion is only internally assessed.

Criterion elements	Rating A	Rating B	Rating C
E1 - Communicates arguments	communicates reasoned mathematical and statistical judgments and arguments using appropriate mathematical terminology and concise language	communicates reasoned mathematical and statistical judgments and arguments using appropriate mathematical terminology and language	communicates mathematical and statistical arguments using appropriate mathematical terminology and language
E2 - Uses mathematical conventions	uses mathematical conventions, systems and constructs including manipulation and use of symbolic expressions, rules and formal systems accurately	uses mathematical conventions, systems and constructs including manipulation and use of symbolic expressions and rules appropriately	uses mathematical conventions, systems and constructs based on definitions and rules

Criterion elements	Rating A	Rating B	Rating C
E3 - Uses units and notation	presents work with correct use of units and notation throughout calculations to convey mathematical information	presents the final answer with correct use of units and notation as required	uses correct units and notation when directed to include them in an answer
E4 - Identifies solution	presents work with the final answer clearly identified and articulated in terms of the questions where necessary.	presents work with the final answer clearly identified.	presents work with the final answer apparent.

Criterion 2: manage and take responsibility for learning and evaluate mathematical development

This criterion is only internally assessed.

Criterion elements	Rating A	Rating B	Rating C
E1 - Reflects on performance	critically reflects upon own learning strengths and weaknesses in order to establish processes used to plan, monitor and assess understanding and performance	analyses own learning strengths and weaknesses in order to establish processes used to plan, monitor and assess understanding and performance	recognises own learning strengths and weaknesses and establishes processes to plan, monitor and assess understanding and performance
E2 - Manages time	monitors and evaluates progress towards meeting own goals and timelines, and plans future actions	monitors and analyses progress towards meeting own goals and timelines	sets own goals and timelines and monitors with support
E3 - Plans and organises	selects and applies effective organisational, planning and self-management skills to manage resources and complete all learning tasks	applies organisational, planning and self-management skills to manage resources and consistently complete tasks	uses a limited range of tools to organise and plan in order to manage resources and complete set tasks

Criterion elements	Rating A	Rating B	Rating C
E4 - Works individually and collaboratively	performs tasks, demonstrates initiative and guides others in their contribution to the completion of individual and collaborative activities	performs tasks and demonstrates initiative when contributing to the completion of individual and collaborative activities	performs tasks as directed to contribute to the completion of individual and collaborative activities
E5 - Monitors task contributions	explains own and other learners' contributions to completion of collaborative activities.	describes own and other learners' contribution to completion of collaborative activities.	identifies own and other learners' contribution to completion of collaborative activities.

Criterion 3: apply mathematical and statistical models to investigate, represent and analyse real-world situations and solve problems*

This criterion is both internally and externally assessed.

Criterion elements	Rating A	Rating B	Rating C
E1 - Makes inferences	explains and analyses problem elements to make informed and logical inferences that can be tested mathematically	identifies and explains problem elements to make logical inferences that can be tested mathematically	identifies problem elements and makes inferences that may be able to be tested mathematically
E2 - Applies mathematical and statistical models to solve problems	selects and applies mathematical and statistical models to solve complex unfamiliar problems in a variety of contexts	selects and applies mathematical and statistical models to solve complex familiar problems	applies mathematical and statistical models to solve simple familiar problems
E3 - Analyses results	relates experimental findings to real-world phenomena, describing differences and analysing possible reasons for these differences	relates experimental findings to real-world phenomena, noting differences and identify possible reasons for these differences	compares experimental findings to expected results in familiar contexts, and identifies possible reasons for differences
E4 - Explains reasonableness of solutions	assesses the reasonableness of results and evaluates the models used to solve complex unfamiliar problems	explains the reasonableness of results and identifies the limitations of models used to solve complex familiar problems	describes the reasonableness of results and identifies the limitations of models used to solve simple familiar problems

Criterion elements	Rating A	Rating B	Rating C
E5 - Draws conclusions	draws valid evidence-based conclusions showing perception and insight that is appropriate to the context.	draws plausible conclusions with supporting evidence that provides some insight appropriate to the context.	draws conclusions that are plausible with limited supporting evidence and detail.

Criterion 4: uses digital technology and other sources to develop mathematical ideas and find solutions to mathematical problems

This criterion is only internally assessed.

Criterion elements	Rating A	Rating B	Rating C
E1 - Accesses and manages information	evaluates authenticity, reliability and validity of information taken from a variety of digital and non-digital sources to develop mathematical ideas	accesses, synthesises and appropriately acknowledges information taken from a variety of digital and non-digital sources to develop mathematical ideas	accesses, manages and acknowledges information from digital and non-digital sources to develop mathematical ideas
E2 - Uses technology to solve problems	explores and applies effective calculator techniques or other digital technologies to solve a range of complex problems in unfamiliar contexts	selects and applies appropriate calculator techniques or other digital technologies to solve a range of simple and complex familiar problems	uses given calculator techniques or other digital technology to solve simple problems
E3 - Uses technology to represent mathematical information	uses digital technologies effectively to move flexibly between different representations of mathematical and statistical information in complex unfamiliar processes	uses digital technologies to graph, display and organise mathematical and statistical information in complex familiar processes	uses digital technologies to graph, display and organise mathematical and statistical information in simple and familiar contexts
E4 - Evaluates technology	interprets and evaluates the inputs and outputs of technology, including critically reflecting on and evaluating the technology used and the outcomes obtained relative to personal, contextual and real-world implications.	identifies and discusses the inputs and outputs of technology and describes how the use of technology can affect outcomes obtained in simple and complex familiar contexts.	identifies and describes how the use of technology can affect outcomes obtained in familiar contexts.

Criterion 5: interpret concepts and apply mathematical techniques to solve problems involving bivariate data analysis and time series analysis using the statistical investigation process*

This criterion is both internally and externally assessed.

Criterion elements	Rating A	Rating B	Rating C
E1 - Represents statistical information and applies associated calculations and techniques	represents statistical information in tables and detailed plots, selects and applies associated techniques and interprets results appropriately in context	represents statistical information in tables and plots, applies associated calculations and techniques and describes results in context	represents statistical information in tables and plots and applies associated calculations and techniques
E2 - Models linear and non-linear relationships from given data	models linear and non-linear relationships from given bivariate data, interprets results and evaluates the appropriateness of the model	models a linear relationship from given bivariate data, interpreting the r and r^2 figures	models a linear relationship from given bivariate data, finding r and r^2
E3 - Interpolates and extrapolates results graphically and algebraically	interpolates and extrapolates results both graphically and algebraically and evaluates the reliability of results, identifying indicators of unreliability	interpolates and extrapolates results both graphically and algebraically and, using a templated approach, discusses the reliability of results	interpolates and extrapolates results both graphically and algebraically
E4 - Interprets, explains and communicates findings	interprets key features of graphs, explains relationships between variables, makes logical inferences based on data and communicates findings in a concise and systematic manner.	interprets key features of graphs, describes relationships between variables, makes predictions based on data and communicates findings.	identifies the key features of graphs and recognises relationships between variables.

Criterion 6: interpret concepts and apply mathematical techniques to model patterns and solve problems involving growth and decay in sequences*

This criterion is both internally and externally assessed.

Criterion elements	Rating A	Rating B	Rating C
E1- Uses arithmetic and geometric sequences	recognises, generates and determines the rule for the n^{th} term of arithmetic and geometric sequences, using this to make predictions	recognises, generates and determines the rule for the n^{th} term of arithmetic and geometric sequences	recognises and generates arithmetic and geometric sequences
E2 - Represents arithmetic and geometric sequences	displays arithmetic and geometric sequences in both tabular and graphical form, using the sequence to model and analyse practical problems involving growth and decay	displays arithmetic and geometric sequences in both tabular and graphical form, using the sequence to model problems involving growth and decay	displays arithmetic and geometric sequences in both tabular and graphical form
E3 - Calculates sum to n terms	calculates the sum of sequences in modelled practical problems	calculates the sum of sequences in complex familiar problems	calculates the sum of sequences in simple familiar problems
E4 - Uses first order linear recurrence relations	uses a first order linear recurrence relation to describe a long-term trend, modelling and analysing practical scenarios.	recognises, generates and uses a first order linear recurrence relation to describe a long-term trend.	recognises and uses a given first order linear recurrence relation.

Criterion 7: interpret concepts and apply mathematical techniques to solve problems involving standard financial models*

This criterion is both internally and externally assessed.

Criterion elements	Rating A	Rating B	Rating C
E1 - Uses growth and decay formula in situations involving compound interest, depreciation and inflation	selects and applies formulas and techniques to calculate, compare, interpret and make informed recommendations in complex unfamiliar situations involving compound interest, depreciation and inflation	selects and applies formulas and techniques to calculate and compare complex familiar situations involving compound interest, depreciation and inflation	applies given formulas and techniques to calculate and solve simple familiar problems involving compound interest, depreciation and inflation
E2 - Uses recurrence relations and to model and solve problems involving compound interest and loans	selects and applies the appropriate recurrence relation to model compound interest loans and investments, investigating the impact of changing interest rates and number of compounding periods	selects and applies the appropriate recurrence relation to model and calculate any variable in complex familiar situations involving compound interest loans and investments	uses a given recurrence relation to model and solve simple familiar problems involving compound interest and loans
E3 - Uses recurrence relations to model and solve problems involving reducing balance loans, annuities and perpetuities	selects and applies the appropriate recurrence relation to model, analyse and solve complex unfamiliar problems involving compound interest loans and investments, annuities and perpetuities	selects and applies the appropriate recurrence relation to model and calculate any variable in complex familiar situations involving reducing balance loans and annuities	uses a given recurrence relation to model and solve simple familiar problems involving reducing balance loans and annuities
E4 - Uses straight line depreciation, unit cost depreciation and reducing balance depreciation	calculates and compares results obtained through straight line depreciation, unit cost depreciation, and reducing balance methods in complex unfamiliar situations.	calculates and compares results obtained through straight line depreciation, unit cost depreciation, and reducing balance methods in complex familiar situations.	applies straight line depreciation and unit cost depreciation methods in simple familiar problems.

Criterion 8: interpret concepts and apply mathematical techniques to represent, analyse and solve practical problems in the two-dimensional plane*

This criterion is both internally and externally assessed.

Criterion elements	Rating A	Rating B	Rating C
E1 - Represents and interprets mathematical information in the two-dimensional plane	interprets mathematical information in complex unfamiliar contexts and creates diagrammatic two-dimensional representations†	interprets mathematical information in complex familiar contexts and creates diagrammatic two-dimensional representations†	represents given mathematical information from simple familiar contexts in the two-dimensional plane
E2 - Applies mathematical techniques to solve problems in the two-dimensional plane	selects and applies appropriate formula‡ to derive mathematical information in order to solve complex unfamiliar problems	selects and applies appropriate formula‡ and mathematical information to solve complex familiar problems	uses given formula‡ and mathematical information to solve simple familiar problems
E3 - Applies knowledge of concepts to solve practical problems and make decisions based on mathematics	applies concepts§ to model, compare and offer optimal solutions to complex unfamiliar situations.	applies basic concepts§ to model, explore compare and solve problems in complex familiar situations.	defines the meaning of key terms and explores practical examples of basic concepts§ in simple familiar situations.

† In applications of trigonometry and Earth geometry, representations include: labelled sketches.

In graphs, networks and decision mathematics representations include: labelled sketches, graphs, networks, adjacency matrix.

‡ In applications of trigonometry and Earth geometry, formula include: Pythagoras' theorem, trigonometric ratios, sine rule, cosine rule, Heron's rule and the spherical cosine rule.

In graphs, networks and decision mathematics formula include: Euler's formula, Prim's algorithm, determining earliest starting time and latest starting time, calculating float times, the Hungarian algorithm.

§ In applications of trigonometry and Earth geometry, concepts range from understanding the meaning of basic ideas, such as great circles and co-ordinated universal time, to angles of latitude and longitude in relation to the perimeter and prime meridian and the link between longitude and time.

In graphs, networks and decision mathematics, concepts range from the basic idea of a weighted graph and network and includes terms such as: graph, edge, vertex, loop, degree of a vertex, subgraphs, bipartite graph, directed graph, arc, planar graph, face, walk, trail, path, closed walk, closed trail, cycle, connected graph, bridge, tree, spanning tree and the concept of critical path analysis, maximum-flow minimum-cut theorem and optimum assignment.

Quality assurance

The following processes will be facilitated by TASC to ensure there is:

- a match between the standards of achievement specified in the course and the skills and knowledge demonstrated by learners
- community confidence in the integrity and meaning of the qualification.

Process – TASC gives course providers feedback about any systematic differences in the relationship of their internal and external assessments and, where appropriate, seeks further evidence through audit and requires corrective action in the future.

Qualifications and award requirements

The final award will be determined by the Office of Tasmanian Assessment, Standards and Certification from 13 ratings, 8 from the internal assessment, 5 from external assessment.

The minimum requirements for an award in this course are as follows:

EXCEPTIONAL ACHIEVEMENT (EA)

10 'A' ratings, 3 'B' ratings, (3 'A' ratings, 2 'B' ratings from external assessment)

HIGH ACHIEVEMENT (HA)

5 'A' ratings, 5 'B' ratings, 3 'C' ratings, (1 'A' ratings, 3 'B' ratings, 1 'C' rating from external assessment)

COMMENDABLE ACHIEVEMENT (CA)

6 'B' ratings, 6 'C' ratings, (2 'B' ratings, 3 'C' ratings from external assessment)

SATISFACTORY ACHIEVEMENT (SA)

11 'C' ratings, (3 'C' ratings from external assessment)

PRELIMINARY ACHIEVEMENT (PA)

6 'C' ratings

A learner who otherwise achieves the ratings for a CA (Commendable achievement) or SA (Satisfactory achievement) award but who fails to show any evidence of achievement in one or more criteria ('z' notation) will be issued with a PA (Preliminary achievement) award.

Course evaluation

Years 9-12 Learning will develop and regularly review and revise the curriculum. Course evaluation is informed by the experience of the course's implementation, delivery and assessment. More information about course evaluation can be found on the Years 11 and 12 website.

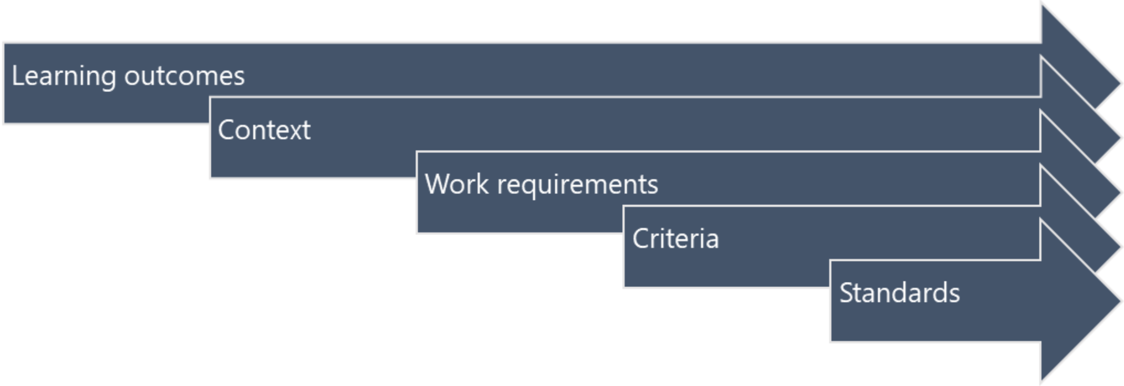
Course developer

This course has been developed by the Department of Education's Years 9-12 Learning Unit in collaboration with Catholic Education Tasmania and Independent Schools Tasmania.

Accreditation and version history

Version 1. Accredited on 14 April 2022 for use from 1 January 2023 to 31 December 2027.

Appendix – Line of sight



Learning outcomes	Course content: module	Work requirements: module	Criterion	Criterion elements	General capabilities
1. communicate arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language	1, 2, 3	1, 2, 3	1	1, 2, 3, 4	
2. plan, organise and manage learning in order to complete tasks and evaluate progress	1, 2, 3	1, 2, 3	2	1, 2, 3, 4, 5	
3. apply modelling, problem solving and mathematical reasoning to interpret, represent and justify the reasonableness of solutions to problems and answers to statistical questions	1	1	3	1, 2, 3, 4, 5	
4. choose and use technology appropriately and effectively	1	1	4	1, 2, 3, 4	
5. understand and apply concepts and techniques of bivariate data analysis and time series analysis using the statistical investigation process	2	2	5	1, 2, 3, 4	
6. understand and apply concepts and techniques to model and solve problems involving growth and decay in sequences	2	2	6	1, 2, 3, 4	
7. understand and apply concepts and techniques to solve problems involving loans, investment and annuities	3	3	7	1, 2, 3, 4	

Learning outcomes	Course content: module	Work requirements: module	Criterion	Criterion elements	General capabilities
8. understand and apply concepts and techniques to represent, analyse and solve problems in the two-dimensional plane	3	3	8	1, 2, 3,	

Appendix 2 – Alignment to curriculum frameworks

Links to Foundation to Year 10:

The proposed *General Mathematics* suite provides students with a breadth of mathematical and statistical experience that encompasses and builds on all three strands of the F–10 curriculum.

For all content areas of *General Mathematics*, the proficiency strands of understanding, fluency, problem solving and reasoning from the F–10 curriculum are still very much applicable and should be inherent in students' learning of the subject. Each strand is essential, and all are mutually reinforcing.

Alignment to Australian Curriculum Senior Secondary Framework:

Almost all content in this course is drawn from the Australian Curriculum Senior Secondary Framework: General Mathematics. The content selected for this course primarily comes from Units 3 and 4. The topics 'Graphs and Networks' from Unit 3 and 'Networks and Decision Mathematics' are coupled together in the course as an elective topic in Module 3. Alternatively, if students engage with the alternative elective option, they will study content from Unit 2 'Applications of trigonometry', which is deemed to be equivalent as the concepts of Heron's rule, the Sine rule and the Cosine rule all extend beyond the content covered in F-10. Further to this, the content from the Essential Mathematics framework Unit 4 'Earth geometry and time zones' along with the inclusion of the spherical Cosine rule are included in this elective topic to further extend the application of trigonometry in this course.

Summary of aligned content:

Module	**Topics**	**Australian Curriculum Framework Source**
Module 1	Mathematical proficiencies of problem solving and reasoning	AC: Mathematics
Module 2	1. Statistical analysis including - bivariate data analysis time series analysis 2. Growth and decay in sequences	 General Mathematics Unit 3 General Mathematics Unit 4 General Mathematics Unit 3
Module 3	1. Loans, investment and annuities 2. a) applications of trigonometry, Earth geometry and time zones b) graphs, networks and decision mathematics	General Mathematics Unit 4 General Mathematics Unit 2 Essential Mathematics Unit 4 General Mathematics Unit 3 General Mathematics Unit 4

Appendix 3 – Work requirements

The work requirements of a course are processes, products or performances that provide a significant demonstration of achievement that is measurable against the course's standards. Work requirements need not be the sole form of assessment for a module.

Module 1 work requirements specifications

Work requirement 1 of 2

Title of work requirement: Statistical investigation

Mode or format: investigation

Description: Learners will engage in a statistical investigation to respond to a specific problem, question, issue or hypothesis evidenced by the collection, analysis, and synthesis of primary or secondary data sets with several variables. The investigation will use investigative practices and mathematical techniques as outlined in the course content of this module, supported by research as appropriate. The investigation should occur over an extended and defined timeframe and allow for individual and group work. The task has three components of increasing complexity:

- the construction, description and interpretation of data plots, including smoothed plots where time series data is used
- the calculation and interpretation of summary statistics, including seasonal indices and their application where time series data is used
- the modelling of linear associations, or trends where time series data is used, including the use of data transformation as appropriate.

Learners' responses to the chosen stimulus will focus on:

- making inferences or hypothesis
- classifying and organising data, including collection if relevant
- interpretation of the constructed data plots
- selection and application of mathematical techniques
- modelling of linear associations or trends
- analysis of association including causation, correlation and identification of outliers and their causes
- communication of drawn conclusions including describing any limitations or assumptions made.

Providers will determine the process for submission and timing of the statistical investigation report. As a guide, it is suggested that the statistical investigation report should be submitted as a single word-processed document of no more than 8 pages, plus appendices, which:

- may include photos of hand-written work, including mathematical calculations, written format with the exception of mathematical calculations
- may include photographs, screenshots or embedded data representations taken from graphical software packages
- includes acknowledgements of information sourced from digital and non-digital sources and raw data as appendices, excluded from page count.

Size: 8-10 hours of dedicated class time in scope and sequence

Timing: will be dependent upon the delivery of module 2, topic 2 in the scope and sequence

External agencies: involvement at teacher discretion

Relevant criteria:

- Criterion 1: E1, 2, 3, 4
- Criterion 2: E1, 2, 3, 4, 5
- Criterion 3: E1, 2, 3, 4, 5
- Criterion 4: E1, 2, 3, 4

- Criterion 5: chosen elements as applicable to the context

Work requirement 2 of 2

Title of work requirement: Mathematical modelling or problem-solving task or both
Mode or format: extended response
Description: Learners will engage in problem-solving or mathematical modelling, or both, of one or more real-world contexts involving content drawn from module 2, topic 1 or module 3, topic 1 or 2. Learners' responses to the chosen stimulus will focus on interpretation of the context, selection and application of mathematical techniques, analysis of results or solutions and communication of drawn conclusions including describing any limitations or assumptions made. In preparation and alongside this task, it is likely that shorter practical activities will be engaged. These are designed to support the depth of understanding and engagement in the extended response.
Providers will determine the process for submission and timing of the mathematical modelling or problem-solving task or both. As a guide, it is suggested that the extended response should be submitted as a single word-processed document of no more than 8 pages which:

- may include photos of hand-written work, including mathematical calculations written format with the exception of mathematical calculations
- may include photographs or embedded diagrams or representations taken from graphical software packages
- includes acknowledgements of information sourced from digital and non-digital sources and raw data as appendices, excluded from page count.

Size: 8-10 hours of dedicated class time in scope and sequence
Timing: will be dependent upon the chosen context of the task and align with delivery of the contextually appropriate content in the scope and sequence
External agencies: involvement at teacher discretion
Relevant criteria:

- Criterion 1: E1, 2, 3, 4
- Criterion 2: E1, 2, 3, 4, 5
- Criterion 3: E1, 2, 3, 4, 5
- Criterion 4: E1, 2, 3, 4
- Criterion 6: chosen elements as applicable to the context
- Criterion 7: chosen elements as applicable to the context
- Criterion 8: chosen elements as applicable to the context

The above work requirements require learners to employ mathematical modelling or problem-solving, or both, processes to investigate open-ended contexts.

Module 2 work requirements specifications

Work requirement 1 of 1

Title of work requirement: Application of concepts and techniques
Mode or format: connected series of short responses
Description: Learners will demonstrate their understanding of concepts and application of techniques to model and solve problems involving growth and decay in sequences and to solve problems involving bivariate data analysis and time series analysis.
Size: 4-6 hours of class time
Timing: spaced assessment opportunities aligned to teaching and learning of specified content as outlined in the scope and sequence
External agencies: involvement at teacher discretion

Relevant criteria:

- Criterion 1: E1, 2, 3, 4
- Criterion 2: E1, 2, 3, 4, 5
- Criterion 5: E1, 2, 3, 4
- Criterion 6: E1, 2, 3, 4

Module 3 work requirements specifications

Work requirement 1 of 1

Title of work requirement: Application of concepts and techniques
Mode or format: connected series of short responses
Description: Learners will demonstrate their understanding of concepts and application of techniques to represent, analyse and solve practical problems in the two-dimensional plane and to model and solve problems involving standard financial models.
Size: 4-6 hours of class time
Timing: spaced assessment opportunities aligned to teaching and learning of specified content as outlined in the scope and sequence
External agencies: involvement at teacher discretion
Relevant criteria:

- Criterion 1: E1, 2, 3, 4
- Criterion 2: E1, 2, 3, 4, 5
- Criterion 7: E1, 2, 3, 4
- Criterion 8: E1, 2, 3

Appendix 4 – General capabilities and cross-curriculum priorities

Learning across the curriculum content, including the cross-curriculum priorities and general capabilities, assists students to achieve the broad learning outcomes defined in the *Alice Springs (Mparntwe) Education Declaration* (December 2019).

General capabilities:

The general capabilities play a significant role in the Australian curriculum in equipping young Australians to live and work successfully in the twenty-first century.

In the Australian curriculum, capability encompasses knowledge, skills, behaviours and dispositions. Students develop capability when they apply knowledge and skills confidently, effectively and appropriately in complex and changing circumstances, in their learning at school and in their lives outside school.

The general capabilities include:

- Critical and creative thinking
- Ethical understanding
- Information and communication technology capability
- Intercultural understanding
- Literacy
- Numeracy
- Personal and social capability

Cross-curriculum priorities:

Cross-curriculum priorities enable students to develop understanding about and address the contemporary issues they face, for their own benefit and for the benefit of Australia as a whole. The priorities provide national, regional and global dimensions which will enrich the curriculum through development of considered and focused content that fits naturally within learning areas. Incorporation of the priorities will encourage conversations between students, teachers and the wider community.

The cross-curriculum priorities include:

- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia's engagement with Asia
- Sustainability

Appendix 5 – Glossary

Term	Definition	Source acknowledgement
adjacency matrix	An adjacency matrix for a non-directed graph with n vertices is a $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j. In an adjacency matrix a loop is counted as one edge. For a directed graph, the entry in row i and column j is the number of directed edges, arcs, joining the vertex i and j in the direction i to j.	ACARA
algorithm	An algorithm is a precisely defined routine procedure that can be applied and systematically followed through to a conclusion.	ACARA
angle of depression	When an observer looks at an object that is lower than the eye of the observer, the angle between the line of sight and the horizontal is called the angle of depression.	ACARA
angle of elevation	When an observer looks at an object that is higher than the eye of the observer, the angle between the line of sight and the horizontal is called the angle of elevation.	ACARA
annuity	An annuity is a compound interest investment from which payments are made on a regular basis for a fixed period of time. At the end of this time the investment has no residual value.	ACARA
arithmetic sequence	A sequence of numbers such that the difference between any two successive members of the sequence is constant. For example, the sequence 2, 5, 8, 11, 14, 17, … is an arithmetic sequence with first term 2 and common difference 3, by inspection of the sequence, the rule for the nth term t_n of this sequence is: $t_n = 2 + (n-1)3 = 3n - 1, n \geq 1$, if t_n is used to denote the nth term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2, t_{n+1} = 3, n \geq 1$	QCAA

Term	Definition	Source acknowledgement
association	A general term used to describe the relationship between two, or more, variables. The term association is often used interchangeably with the term correlation. The latter tends to be used when referring to the strength of a linear relationship between two numerical variables.	ACARA
average percentage method	In the average percentage method for calculating a seasonal index, the data for each 'season' are expressed as percentages of the average for the year. The percentages for the corresponding 'seasons' for different years are then averaged using a mean or median to arrive at a seasonal index.	ACARA
bearing	The direction of a fixed point, or the path of an object, from the point of observation.	QCAA
bipartite graph	A bipartite graph is a graph whose set of vertices can be split into two distinct groups in such a way that each edge of the graph joins a vertex in the first group to a vertex in the second group.	ACARA
book value	The book value is the value of an asset recorded on a balance sheet. The book value is based on the original cost of the asset less depreciation. There are three commonly used methods for calculating yearly depreciation in the value of an asset; namely, reducing balance depreciation, flat rate depreciation or unit cost depreciation.	ACARA
break-even point	The break-even point is the point at which revenue begins to exceed the cost of production.	ACARA
categorical data	Data associated with a categorical variable is called categorical data.	ACARA

Term	Definition	Source acknowledgement
categorical variable	A categorical variable is a variable whose values are categories. Examples include blood group (A, B, AB or O) or house construction type, brick, concrete, timber, steel, other. Categories may have numerical labels; for example, the numbers worn by player in a sporting team, but these labels have no numerical significance, they merely serve as labels.	ACARA
causal relationship	A relationship between an explanatory and a response variable is said to be causal if the change in the explanatory variable actually causes a change in the response variable. Simply knowing that two variables are associated, no matter how strongly, is not sufficient evidence by itself to conclude that the two variables are causally related.	QCAA
coefficient of determination	In a linear model between two variables, the coefficient of determination (R2) is the proportion of the total variation that can be explained by the linear relationship existing between the two variables, usually expressed as a percentage. For two variables only, the coefficient of determination is numerically equal to the square of the correlation coefficient (r2).	ACARA
common response	A response is said to be common when there is no causation, but instead the association is explained by at least one other variable that is associated with both the explanatory and the response variable.	ACARA
compass bearings	Compass bearings are specified as angles either side of north or south, that describe the direction of a fixed point, or the path of an object. For example, a compass bearing of N50°E is found by facing north and moving through an angle of 50° towards east.	ACARA
complete graph	A complete graph is a simple graph in which every vertex is joined to every other vertex by an edge. The complete graph with n vertices is denoted Kn.	ACARA

Term	Definition	Source acknowledgement
compound interest	The interest earned when each successive interest payment is added to the principal for the purpose of calculating the next interest payment. For example, if the principal (P) earns compound interest (A) at the interest rate (i) expressed as a percentage per period, then after (n) compounding periods the total amount accrued is: $$A = P(1 + i)^n$$ When plotted on a graph, the total amount accrued is shown to grow exponentially.	QCAA
confounding	Confounding exists in situations where there may be causation, but the change may also be caused by one or more uncontrolled variables whose effects cannot be disentangled from the effect of the response variable.	ACARA
connected graph	A graph is connected if there is a path between each pair of vertices. A bridge is an edge in a connected graph that, if removed, leaves a graph disconnected.	ACARA
Consumer Price Index	The Consumer price index (CPI) is a measure of changes, over time, in retail prices of a constant basket of goods and services representative of consumption expenditure by resident households in Australian metropolitan areas.	ACARA
continuous data	Data associated with a continuous variable is called continuous data.	ACARA
continuous variable	A continuous variable is a numerical variable that can take any value that lies within an interval. In practice, the values taken are subject to accuracy of the measurement instrument used to obtain these values. Examples include height, reaction time, temperature and systolic blood pressure.	ACARA
correlation	Correlation is a measure of the strength of the linear relationship between two variables.	ACARA

Term	Definition	Source acknowledgement
correlation coefficient	The correlation coefficient (r) is a measure of the strength of the liner relationship between a pair of variables.	ACARA
cosine ratio	In any right-angled triangle, $$\cos\theta = \frac{adjacent}{hypotenuse}$$	QCAA
cosine rule	For a triangle of side lengths a, b and c and angles A, B and C, the cosine rule states that: $$c^2 = a^2 + b^2 - 2ab \times \cos C$$	ACARA
critical path analysis (CPA)	A project often involves many related activities, some of which cannot be started until one or more earlier tasks have been completed. One way of scheduling such activities that takes this into account is to construct a network diagram. Critical path analysis is a method for determining the longest path, the critical path, in such a network and hence the minimum time in which the project can be completed. There may be more than one critical path in the network.	ACARA
cut - in a flow network	In a flow network, a cut is a partition of the vertices of a graph into two separate groups with the source in one group and the sink in the other. The capacity of the cut is the sum of the capacities of the cut edges directed from source to sink. Cut edges directed from sink to source are ignored.	ACARA
cycle	A cycle is a closed walk that begins and ends at the same vertex and in which has no repeated edges or vertices except the first.	ACARA
degree of a vertex	In a graph, the degree of a vertex is the number of edges incident with the vertex, with loops counted twice. It is denoted deg v.	ACARA

Term	Definition	Source acknowledgement
directed graph- digraph	A directed graph is a diagram comprising points, called vertices, joined by directed lines called arcs. The directed graphs are commonly called digraphs.	ACARA
discrete data	Discrete data is data associated with a discrete variable. Discrete data is sometimes called count data.	ACARA
discrete variable	A discrete variable is a numerical variable that can take only integer values. Examples include the number of people in a car, the number of decayed teeth in an 18-year-old male etc.	ACARA
earliest starting time (EST)	Using a forward scan of a network diagram to determine the earliest time an activity can begin.	QCAA
edge	In a graph, an edge is a line that connects tow vertices.	ACARA
effective annual rate of interest	The effective annual rate of interest $i_{effective}$ is used to compare the interest paid on loans, or investments, with the same nominal annual interest rate i but with different compounding periods, daily, fortnightly, monthly, quarterly, annually, other. If the number of compounding periods per annum is n, then: $$i_{effective} = (1 + \frac{i}{n})^n - 1$$	ACARA
Euler's rule	For a connected planar graph, Euler's rule states that: $v + f - e = 2$ where v is the number vertices, e the number of edges and f is the number of faces.	ACARA

Term	Definition	Source acknowledgement
Eulerian graph	A connected graph is Eularian if it has a closed trail, starts and ends at the same vertex, that is, includes every edge once only; such a trail is called a Eulerian trail. An Eularian trail may include repeated vertices. A connected graph is semi-Eularian if there is an open trail that includes every edge once only.	ACARA
explanatory variable	When investigating relationships in bivariate data, the explanatory variable, independent variable, is the variable used to explain or predict a difference in the response variable, dependent variable; for example, when investigating the relationship between the temperature of a loaf of bread and the time it has spent in a hot oven, temperature is the response variable and time is the explanatory variable.	ACARA
extrapolation	In the context of fitting a linear relationship between two variables, extrapolation occurs when the fitted model is used to make predictions using values of the explanatory variable that are outside the range of the original data. Extrapolation is a dangerous process as it can sometimes lead to quite erroneous predictions.	ACARA
face	The faces of a planar graph are the regions bounded by the edges including the outer infinitely large region.	ACARA
first-order linear recurrence relation	A first-order linear recurrence relation is defined by the rule: $t_0 = a, t_{n+1} = bt_n + c$, for $n \geq 1$ For example, the rule $t_0 = 10, t_n = 5t_{n-1} + 1$, for $n \geq 1$ is a first-order recurrence relation. The sequence generated by this rule starting at t_0 is: 10, 51, 256, … as shown below. $t_0 = 10, t_1 = 5t_0 + 1 = 5 \times 10 + 1 = 51,$ $t_2 = 5t_1 + 1 = 5 \times 51 + 1 = 256, \ldots$	ACARA

Term	Definition	Source acknowledgement
flat rate depreciation	In flat rate or straight-line depreciation, the value of an asset is depreciated by a fixed amount each year. Usually, this amount is specified as a fixed percentage of the original cost.	ACARA
float time	Is the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks. All activities on a critical path have zero floats.	ACARA
flow network	A flow network is a directed graph where each edge has a capacity, e.g. 100 cars per hour, 800 litres per minute etc., and each edge receives a flow. The amount of flow on an edge cannot exceed the capacity of the edge. A flow must satisfy the restriction that the amount of flow into a node equals the amount of flow out of it, except when it is a source, which has more outgoing flow, or a sink, which has more incoming flow. A flow network can be used to model traffic in a road system, fluids in pipes, currents in an electrical circuit, or any situation in which something travels through a network of nodes.	ACARA
food web	A food web, or food chain, depicts feeding connections, who eats whom, in an ecological community.	ACARA
geometric growth or decay	A sequence displays geometric growth or decay when each term is some constant multiple, greater or less than one, of the preceding term: a multiple greater than one corresponds to growth, a multiple less than one corresponds to decay, e.g. 1, 2, 4, … displays geometric growth because each term is double the previous term, 100, 10, 0.1, … displays geometric decay because each term is one tenth of the previous term.	QCAA

Term	Definition	Source acknowledgement
geometric sequence	A sequence of numbers where each term after the first is found by multiplying the previous term by a fixed non-zero number (excluding ±1) called the common ratio, e.g. 2, 6, 18, ... is a geometric sequence with first term 2 and common ratio 3; by inspection of the sequence, the rule for the nth term of this sequence is: $t_n = 2 \times 3^{(n-1)}, n \geq 1$ if t_n is used to denote the nth term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2, t_n + 1 = 3t_n, n \geq 1$	QCAA
Goods and Services Tax (GST)	The Goods and Services Tax (GST) is a broad sales tax of 10% on most goods and services and other items sold or consumed in Australia.	QCAA
gradient	The gradient of a line describes its steepness, incline, or grade. Gradient is normally described by the ratio of the "rise" divided by the "run" between two points on a line.	ACARA
graph	A graph is a diagram that consists of a set of points, called vertices, that are joined by a set of lines called edges. Each edge joins two vertices. A loop is an edge in a graph that joins a vertex in a graph to itself. Two vertices are adjacent if they a joined by an edge. Two or more edges connect the same vertices are called multiple edges.	ACARA
Hamiltonian cycle	A Hamiltonian cycle is a cycle that includes each vertex in a graph, except the first, once only.	ACARA
Hamiltonian path	A Hamiltonian path is path that includes every vertex in a graph once only. A Hamiltonian path that begins and ends at the same vertex is a Hamiltonian cycle.	ACARA

Term	Definition	Source acknowledgement
Heron's rule	Heron's rule is a rule for determining the area of a triangle given the length of its sides. The area A of a triangle of side lengths a, b and c is given by: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$	ACARA
Hungarian algorithm	The Hungarian algorithm is used to solve assignment, allocation, problems.	ACARA
interpolation	In the context of fitting a linear relationship between two variables, interpolation occurs when the fitted model is used to make predictions using values of the explanatory variable that lie within the range of the original data.	ACARA
irregular variation or noise, time series	Irregular variation or noise is erratic and short---term variation in a time series that is the product of chance occurrences.	ACARA
latest starting time (LST)	Using a backward scan of a network diagram to determine the latest time an activity can begin.	ACARA
least-squares line	In fitting a straight---line $y = a + bx$ to the relationship between a response variable y and an explanatory variable x, the least---squares line is the line for which the sum of the squared residuals is the smallest.	ACARA

Term	Definition	Source acknowledgement
linear growth or decay, sequence	A sequence displays linear growth or decay when the difference between successive terms is constant. A positive constant difference corresponds to linear growth while a negative constant difference corresponds to decay. Examples: The sequence, 1, 4, 7, … displays linear growth because the difference between successive terms is 3. The sequence, 100, 90, 80, … displays linear decay because the difference between successive terms is –10. By definition, arithmetic sequences display linear growth or decay.	ACARA
length of a walk	The length of a walk is the number of edges it includes.	ACARA
mean	The arithmetic mean, $\bar{x}$, of a list of numbers is the sum of the data values divided by the number of values in the list. In everyday language, the arithmetic mean is commonly called the average.	ACARA
median	The median is the value in a set of ordered set of data values that divides the data into two parts of equal size. When there are an odd number of data values, the median is the middle value. When there is an even number of data values, the median is the arithmetic mean of the two central values.	ACARA
minimum cut-maximum flow theorem	The minimum cut-maximum flow theorem states that in a flow network, the maximum flow from the source to the sink is equal to the capacity of the minimum cut.	ACARA
mode	The mode is the most frequently occurring value in a data set.	ACARA
moving average	In a time series, a simple moving average is a method used to smooth the time series whereby each observation is replaced by a simple average of the observation and its near neighbours. This process reduces the effect of non-typical data and makes the overall trend easier to see.	ACARA

Term	Definition	Source acknowledgement
network	The word network is frequently used in everyday life, e.g. television network, rail network, social network etc. Weighted graphs or digraphs can often be used to model such networks.	ACARA
outlier	An outlier in a set of data is an observation that appears to be inconsistent with the remainder of that set of data. An outlier is a surprising observation.	ACARA
path in a graph	A path in a graph is a walk in which all of the edges and all the vertices are different. A path that starts and finishes at different vertices is said to be open, while a path that starts and finishes at the same vertex is said to be closed. A cycle is a closed path.	ACARA
perpetuity	A perpetuity is a compound interest investment from which payments are made on a regular basis in perpetuity, forever. This is possible because the payments made at the end of each period exactly equal the interest earned during that period.	ACARA
planar graph	A planar graph is a graph that can be drawn in the plane. A planar graph can always be drawn so that no two edges cross.	ACARA
price to earnings ratio of a share	The price to earnings ratio of a share (P/E ratio) is defined as: $$\frac{P}{E} ratio = \frac{Market\ price\ per\ share}{Annual\ earnings\ per\ share}$$	ACARA
Prim's algorithm	An algorithm for determining a minimum spanning tree in a connected weighted graph.	ACARA
Pythagoras' theorem	The square of the hypotenuse of a right-angled triangle equals the sum of the squares of the lengths of the other two sides. As a rule: $c^2 = a^2 + b^2$, where c is the length of the hypotenuse.	ACARA
reasoned argument or conclusion	One that is sound, well-groomed, considered and thought out	

Term	Definition	Source acknowledgement
recurrence relation	A recurrence relation is an equation that recursively defines a sequence; that is, once one or more initial terms are given, each further term of the sequence is defined as a function of the preceding terms.	ACARA
recursion	The repeated application of a recursive procedure or definition.	Oxford Dictionary
reducing balance depreciation	In reducing balance depreciation, the value of an asset is depreciated by a fixed percentage of its value each year, until the asset has no residual value after a defined number of years. Reducing balance depreciation is sometimes called diminishing value depreciation.	ACARA
reducing balance loan	A reducing balance loan is a compound interest loan where the loan is repaid by making regular payments and the interest paid is calculated on the amount still owing, the reducing balance of loan, after each payment is made.	ACARA
residual values	The difference between the observed value and the value predicted by a statistical model; for example, by a least-squares line.	ACARA
residual plot	A residual plot is a scatterplot with the residual values shown on the vertical axis and the explanatory variable shown on the horizontal axis. Residual plots are useful in assessing the fit of the statistical model; for example, by a least-squares line.	ACARA
response variable	Also known as the dependent variable; its value is dependent on the value of the explanatory, or independent, variable.	QCAA
round-robin sporting competition	A single round robin sporting competition is a competition in which each competitor plays each other competitor once only.	ACARA

Term	Definition	Source acknowledgement
scale factor	A number that scales, or multiplies, some quantity. In the equation $y = kx$, k is the scale factor for x; if two or more figures are similar, their sizes can be compared. The scale factor is the ratio of the length of one side on one figure to the length of the corresponding side on the other figure. It is a measure of magnification; the change of size.	ACARA
scatterplot	A two-dimensional data plot using Cartesian co-ordinates to display the values of two variables in a bivariate data set.	ACARA
seasonal adjustment	A term used to describe a time series from which periodic variations due to seasonal effects have been removed.	ACARA
seasonal index, indices	The seasonal index can be used to remove seasonality from data. An index value is attached to each period of the time series within a year. For the seasons of the year, Summer, Autumn, Winter, Spring, there are four separate seasonal indices; for months, there are 12 separate seasonal indices, one for each month, and so on. There are several methods for determining seasonal indices.	ACARA
seasonal variation	A regular rise and fall in the time series that recurs each year. Seasonal variation is measured in terms of a seasonal index.	ACARA
sequence	A sequence is an ordered list of numbers, or objects. For example, 1, 3, 5, 7 is a sequence of numbers that differs from the sequence 3, 1, 7, 5 as order matters. A sequence maybe finite; for example, 1, 3, 5, 7, the sequence of the first four odd numbers, or infinite; for example, 1, 3, 5, … the sequence of all odd numbers.	ACARA
similar figures	Two geometric figures are similar if they are of the same shape but not necessarily of the same size.	QCAA
simple graph	A simple graph has no loops or multiple edges.	ACARA

Term	Definition	Source acknowledgement
sine ratio	In any right-angled triangle, $$\sin\theta = \frac{opposite}{hypotenuse}$$	QCAA
sine rule	For a triangle of side lengths a, b and c and angles A, B and C, the sine rule states that: $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$	ACARA
simple interest	Simple interest is the interest (I) accumulated when the interest payment in each period is a fixed fraction of the principal, e.g. if the principle P earns simple interest at the rate (R) expressed as a percentage per period, then after (T) periods the accumulated simple interest is: $$I = PRT$$ When plotted on a graph, the total amount accrued is shown to grow linearly.	QCAA
spanning tree	A spanning tree is a subgraph of a connected graph that connects all vertices and is also a tree.	ACARA
standard deviation	The standard deviation is a measure of the variability or spread of a data set. It gives an indication of the degree to which the individual data values are spread around their mean. The standard deviation of n observations $x_1, x_2, \dots, x_n$ is: $$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$	QCAA
tangent ratio	In any right-angled triangle, $$\tan\theta = \frac{opposite}{adjacent}$$	QCAA

Term	Definition	Source acknowledgement
time series	Values of a variable recorded, usually at regular intervals, over a period of time. The observed movement and fluctuations of many such series comprise long---term trend, seasonal variation, and irregular variation or noise.	ACARA
time series plot	The graph of a time series with time plotted on the horizontal axis.	ACARA
Trend, time series	Trend is the term used to describe the general direction of a time series, increasing or decreasing, over a long period of time.	ACARA
trail	A trail is a walk in which no edge is repeated.	ACARA
tree	A tree is a connected graph with no cycles.	ACARA
true bearings	True bearings are measured in degrees in a clockwise direction from the north line. Three figures are used to specify the direction. Thus, north is specified as 000°T, east is specified as 090°T, south-east is specified as 135°T.	ACARA
two-way frequency table	A two-way frequency table is commonly used for displaying the two-way frequency distribution that arises when a group of individuals or objects are categorised according to two criteria. The row and column totals represent the total number of observations in each row and column and are sometimes called row sums or column sums. If the table is 'percentaged' using row sums the resulting percentages are called row percentages. If the table is 'percentaged' using column sums the resulting percentages are called column percentages.	ACARA
unit cost depreciation	In unit cost depreciation, the value of an asset is depreciated by an amount related to the number of units produced by the asset during the year.	ACARA

Term	Definition	Source acknowledgement
walk in a graph	A walk in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. A walk that starts and finishes at different vertices is said to be an open walk. A walk that starts and finishes at the same vertex is said to be closed walk.	ACARA
weighted graph	A weighted graph is a graph in which each edge is labelled with a number used to represent some quantity associated with the edge. For example, if the vertices represent towns, the weights on the edges may represent the distances in kilometres between the towns.	ACARA

Appendix 6 – Degree of difficulty of problems

Acknowledgement: The following material has been sourced, with approval, from the Queensland Curriculum and Assessment Authority curriculum.

Within this course, the degree of difficulty of problems a learner can answer correctly is a defining feature of their understanding. Within the criteria and standards, the expected depth of knowledge is described using the following terms.

Simple familiar

Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:

- relationships and interactions are obvious and have few elements; and
- all of the information to solve the problem is identifiable; that is
  - the required procedure is clear from the way the problem is posed, or
  - in a context that has been a focus of prior learning.

Complex familiar

Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:

- relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and
- all of the information to solve the problem is identifiable; that is
  - the required procedure is clear from the way the problem is posed, or
  - in a context that has been a focus of prior learning.

Some interpretation, clarification and analysis will be required to develop responses.

Complex unfamiliar

Problems of this degree of difficulty require students to demonstrate knowledge and understanding of the subject matter and application of skills in a situation where:

- relationships and interactions have a number of elements, such that connections are made with subject matter within and/or across the domains of mathematics; and
- all the information to solve the problem is not immediately identifiable; that is
  - the required procedure is not clear from the way the problem is posed, and
  - in a context in which students have had limited prior experience.

Students interpret, clarify and analyse problems to develop responses.